The Other ADM Result

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FINITE SELF-ENERGY OF CLASSICAL POINT PARTICLES

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The infinite mass self-energy difficulties of quantum field theory already occur, as is well-known, in the corresponding classical theories. Although cutoffs may be introduced to effect renormalization in both the classical and quantum cases, such procedures are physically unsatisfactory. We wish to point out in this note that at least for the static (Coulomb-type) contribution, one obtains finite results for the classical self-energies if the gravitational contribution to the total energy is included. Furthermore, it will

scalar density,[4] i.e., $\int \delta^3(\vec{r})d^3r = 1$. The solution of Eq. (2) which is asymptotically flat is seen to be

$$\chi(r) = 1 + m_0/[32\pi r\chi(0)]. \tag{3}$$

The parameter $m = m_0/\chi(0)$ is given in terms of m_0 by

$$m = \lim_{\epsilon \to 0} 2m_0[1 + (1 + m_0/8\pi\epsilon)^{1/2}]^{-1}. \tag{4}$$

In Eq. (4), ϵ is essentially the "radius" of the

1 Loop ∞'s of "Sub-Gravity" + Matter

- $+\varphi$ ➔ 't Hooft & Veltman (1974)
- $+A_\mu$ ➔ Deser & van N. (1974)
- $+\Psi$ ➔ Deser & van N. (1974)
- $+A_{a\mu}$ ➔ Deser, Tseng & van N. (1974)
- ∂^4 counterterms would renormalize . . .
  . . . but unstable ➔ Stelle (1977)

Conspiracy of Four Principles

1. Continuum Field Theory ➔ ∞ Modes
2. Q. Mechanics ➔ Can't have $q_0=p_0=0$
   - Each mode has $\frac{1}{2}\hbar\omega$ + interactions
3. General Relativity ➔ Energy gravitates
4. Pert. Theory ➔ $\hbar\omega$'s add at 1st order

Maybe Perturbation Theory Gives Wrong Asymptotic Exp.

- What we want:
  $[\text{Tree}]\ \{1 + \#\ (GE^2/\hbar c^5) + \ldots\}$
- What perturbation theory gives:
  $[\text{Tree}]\ \{1 + \ln(\infty)\ (GE^2/\hbar c^5) + \ldots\}$
- Same as if correct series were:
  $[\text{Tree}]\ \{1 + \#\ \ln(GE^2/\hbar c^5)\ (GE^2/\hbar c^5) + \ldots\}$

Eg. Statistical Mechanics of Noninteracting Bosons

- Z(T,V,N) for K = $[m^2c^4 + p^2c^2]^{1/2} - mc^2$

$$Z = \int \frac{d^3x d^3p}{(2\pi\hbar)^3} e^{-K/k_B T} ,$$
$$= \frac{V}{2\pi^2\hbar^3c^3} \int_0^\infty dK\, e^{-\beta K}(K + mc^2)\sqrt{K^2 + 2Kmc^2} .$$

- Ln[Ξ(T,V,μ)] for K = $p^2/2m$

$$\ln(\Xi) = \int \frac{d^3x d^3p}{(2\pi\hbar)^3} \ln\Big[\sum_{n=0}^{\infty} e^{-n\beta(K-\mu)}\Big] ,$$
$$= \frac{2V}{\sqrt{\pi}}\Big(\frac{m}{2\pi\hbar^2}\Big)^3 \int_0^\infty dK\, K^{\frac{1}{2}} \ln\Big[\frac{1}{1 - e^{-\beta(K-\mu)}}\Big] ,$$
$$= V\Big(\frac{mk_BT}{2\pi\hbar^2}\Big)^3 \sum_{k=1}^{\infty} k^{-\frac{5}{2}} e^{k\beta\mu} .$$

Expanding Z for $x = mc^2/k_BT << 1$

- $t = K/k_BT$

$$Z = \frac{V}{2\pi^2}(\frac{k_BT}{\hbar c})^3 \int_0^\infty dt\, e^{-t}\, t^2(1+\frac{x}{t})\sqrt{1+\frac{2x}{t}}$$

- Wrong: 0 at x^3 + ∞ at x^4

$$Z = \frac{V}{2\pi^2}(\frac{k_BT}{\hbar c})^3 \int_0^\infty dt\, e^{-t}\, t^2\left\{1+2\cdot\frac{x}{t}+\frac{1}{2}\cdot\frac{x^2}{t^2}+0\cdot\frac{x^3}{t^3}+\frac{1}{8}\cdot\frac{x^4}{t^4}\ldots\right\}$$

- Right: $\neq 0$ at x^3 + $x^4 \ln(x)$

$$Z = \frac{V}{2\pi^2}(\frac{k_BT}{\hbar c})^3\left\{2+2x+\frac{1}{2}x^2-\frac{1}{6}x^3-\frac{1}{48}x^4\ln(x)+\ldots\right\}$$

Expanding ln(Ξ) for $-\beta\mu$ = x << 1

$$\ln(\Xi) = V n_Q f(x) \equiv V n_Q \sum_{k=1}^{\infty} k^{-\frac{5}{2}} e^{-kx}$$

- Wrong: f(x) = $\zeta(5/2) - \zeta(3/2)\, x + \infty\, x^2$

$$f(x) = \sum_{k=1}^{\infty} \left\{ k^{-\frac{5}{2}} - k^{-\frac{3}{2}} x + \frac{1}{2} k^{-\frac{1}{2}} x^2 + \ldots \right\}$$

- Right: f(x) = (Same) + $4\pi^{1/2}/3\; x^{3/2}$ +. . .

$$f''(x) = \sum_{k=1}^{\infty} k^{-\frac{1}{2}} e^{-kx} \approx \int_0^{\infty} dk\, k^{-\frac{1}{2}} e^{-kx} = \left(\frac{\pi}{x}\right)^{\frac{1}{2}}$$

- 2nd order IS small, just not $\sim x^2$

Charged shell of radius R → 0 (ADM 1960)

- Without GR: $mc^2 = m_0c^2 + q^2/8\pi\epsilon_o R$

  "renormalize" with $m_0c^2 = m_{obs}c^2 - q^2/8\pi\epsilon_o R$
- With GR: $mc^2 = m_0c^2 + q^2/8\pi\epsilon_o R - Gm^2/2R$

$$m = \frac{Rc^2}{G}\left[-1 + \sqrt{1 + \frac{2Gm_0}{Rc^2} + \frac{Gq^2}{4\pi\epsilon_0 R^2c^2}}\right] \longrightarrow \sqrt{\frac{q^2}{4\pi\epsilon_0 G}}$$

- Perturbative Result:

  ➔ Oscillating series of ever-higher ∞'s

Lessons from the ADM Result

- Gravity might well cancel ∞'s
- But not perturbatively, eg $m = \alpha^{1/2} m_{Pl}$
  - Not analytic in α
  - Diverges for $G \to 0$
- "Perturbative Conundrum": Grav. response always an order behind
- Hopeless to compute exactly ➔ Seek new expansion in which "gravity can keep up"

Past Efforts

- Bryce DeWitt
  - PRL 13 (1964) 114-118
- Isham, Salam & Strathdee
  - PRD3 (1971) 1805-1817
  - PRD5 (1972) 2548-2565
- Mike Duff
  - PRD4 (1971) 1851-1855 (+ Huskins & Rothery)
  - PRD7 (1973) 2317-2326
  - PRD9 (1974) 1837-1839

Mass from the Propagator

- $|k,\alpha>$ has k^i & $\omega_\alpha = [k^2 + \alpha^2]^{1/2}$
- Kallen-like Rep. for $<\Omega|\varphi(x)\varphi^*(y)|\Omega>$

$$\sum_\alpha \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_\alpha} \langle \Omega|\varphi(x)|\alpha\rangle\langle\alpha|\varphi^*(y)|\Omega\rangle$$

$$= \sum_\alpha \int \frac{d^3k}{(2\pi)^3} \frac{Z(\alpha)}{2\omega_\alpha} e^{-i\omega_\alpha(x^0-y^0)} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}$$

- $M = \lim[x^0 \to +\infty, y^0 \to -\infty] \quad i/(x^0-y^0) \times$

$$\times \ln\left[\int d^3x \, \langle\Omega|\varphi(x)\varphi^*(y)|\Omega\rangle\right]$$

Integrate Matter Out

- S[g,A] = ∫d⁴x $\mathcal{L}$

$$\mathcal{L} = \frac{1}{16\pi G} R\sqrt{-g} - \frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g}$$

- S[g,A,φ*,φ] = ∫d⁴x φ*$\mathcal{D}$[g,A]φ

$$\mathcal{D}[g,A] = (\partial_\mu + ieA_\mu)[\sqrt{-g} g^{\mu\nu} (\partial_\nu + ieA_\nu)] - m^2 \sqrt{-g}$$

- <Ω|T[φ(x)φ*(y)]|Ω>

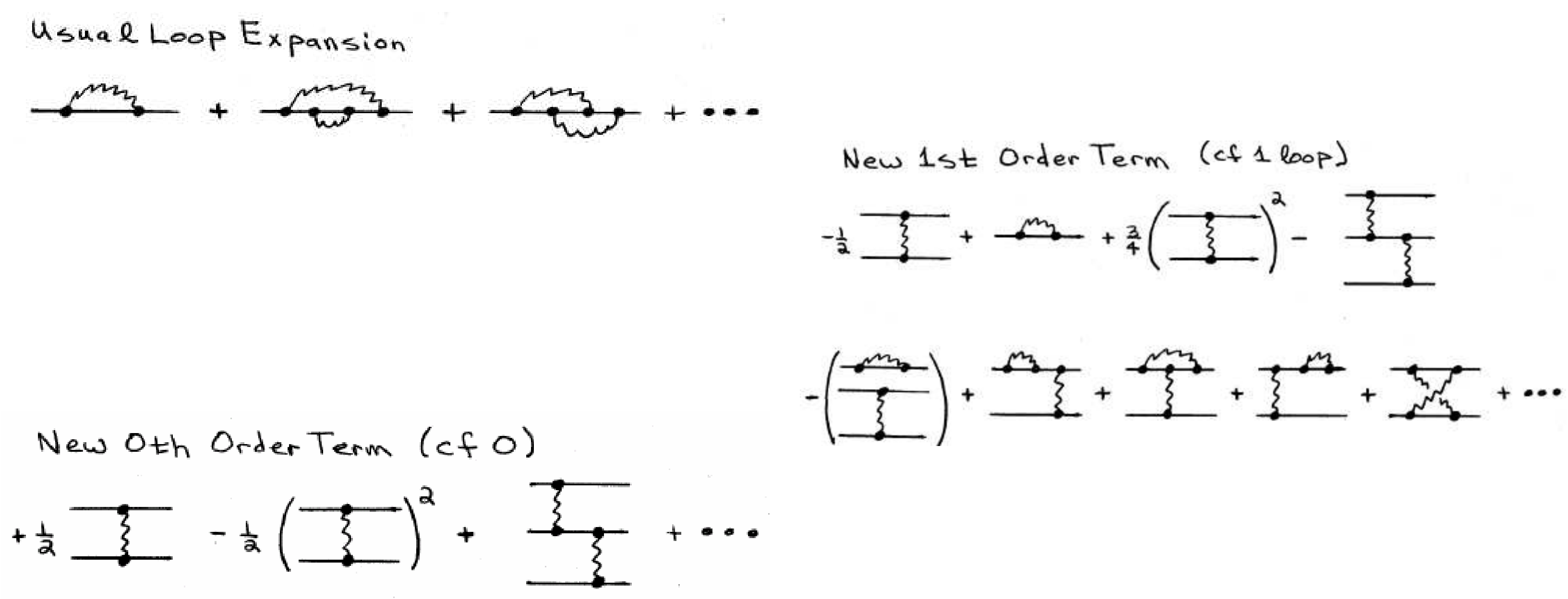
$$= \int [dg][dA]\, e^{iS[g,A]} \frac{\langle x | i\mathcal{D}^{-1}[g,A] | y \rangle}{\det(\mathcal{D}[g,A])}$$

A Different Expansion

- Stationary phase, but include
  $\langle x|i\mathcal{D}^{-1}[g,A]|y\rangle$ with S[g,A]
- Doesn’t sum classes of loops
- Adds 0 = $\pm$ (New Diagrams)
- Cf. perturbative comparison for

$$2me^{im(x^0-y^0)}\int d^3x\,\langle x|i\mathcal{D}^{-1}[g,A]|y\rangle = 1+\ldots$$

Comparing Diagrams for $\ln[2me^{im\Delta t}\int d^3x \langle \varphi(x)\varphi^*(y)\rangle]$



Physical Interpretation: A QM Particle Causing Its Own Fields

- Recall free result for $x^0 > y^0$

$$i\Delta(x;y) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_m}} e^{-i\omega_m x^0 + i\vec{k}\cdot\vec{x}} \times \frac{1}{\sqrt{2\omega_m}} e^{i\omega_m y^0 - i\vec{k}\cdot\vec{y}}$$

- $\int dx^3$ selects for $k^i = 0$
- Generally u[g,A](x) $\ni \mathcal{D}$u = 0

$$\langle x | i\mathcal{D}^{-1}[g,A] | y\rangle = \sum u[g,A](x) \times u^*[g,A](y)$$

- 0^{th} Order
  - u[g,A](x) moves in $g_{\mu\nu}$ & A_μ
  - u[g,A](x) <u>sources</u> $g_{\mu\nu}$ & A_μ

Looking for Bound States

- Two Cases:
  - No bound states ➔ hard scat. problem
  - Bound states ➔ lowest one dominates
- Simplifications
  - $g_{\mu\nu}dx^{\mu}dx^{\nu} = -A(r)dt^2 + B(r)dr^2 + r^2d\Omega$
  - $A_{\mu}dx^{\mu} = \Phi(r)dt$
  - Use variational techniques to bound
- What if there is more than one?

What about Fermions?

- Same representation works for fermions
  - Don't need to drop $(\varphi^*\varphi)^2$ term
- NB fermion kinetic operators give <u>bosonic</u> QM problem
- And fermions have <u>spin</u>

Potential Significance of Spin

- Parts of $\Psi(x)$ seen thru BIG γ factors
- Spinning disk model

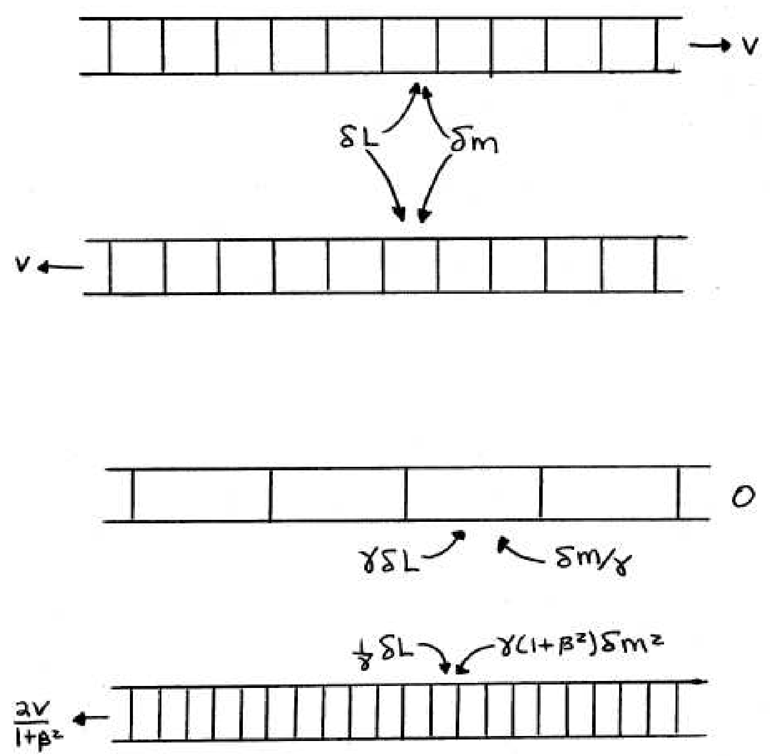
  $\frac{1}{2}\hbar = \frac{1}{2} m R^2 \omega$

  $\rightarrow R\omega = \hbar / mR$

  $R = \hbar / mc \rightarrow R\omega = c$

- Cons. Parallel Strips
- Top strip rest frame

  $R_S = \gamma(1+\beta^2)\, G\delta m / c^2$



Conclusions

- QGR ∞'s may be perturbative artifacts
- This isn't crazy
  - Physics is reasonable
  - Many examples from simple physics
- But it IS hard to check
- Hopeless to compute exactly
  - Need alternate expansion
  - One example would suffice!

Our Program: Generalize the Other ADM Result to QFT

- 0^{th} order: QM particle in fields it causes
- Gauge invariant
- Adds 0 = (New – New) to loop expansion
- Breaks perturbative conundrum
  - Normal QGR response always an order behind
- Interesting case is bound states
  - What are they if more than one?
- Should be solvable numerically