The Classical Universes of the No-Boundary Quantum State

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A Quantum Universe

If the universe is a quantum mechanical system it has a quantum state. What is it?

$$\Psi$$

That is the problem of Quantum Cosmology.

No State --- No Predictions

- The probability p at time t of an alternative represented by a projection P(t) (e.g a range of position) in a state $|\Psi\rangle$ is:

$$p = ||P(t)|\Psi\rangle||^2$$

$$P(t) = e^{iHt/\hbar}P(0)e^{-iHt/\hbar}$$

- If we don't have the operator P and H and the state $|\Psi\rangle$ there are no probabilities and no predictions.

Ignorance is not Bliss

Ignorance of the state means: $\rho = I/Tr(I)$

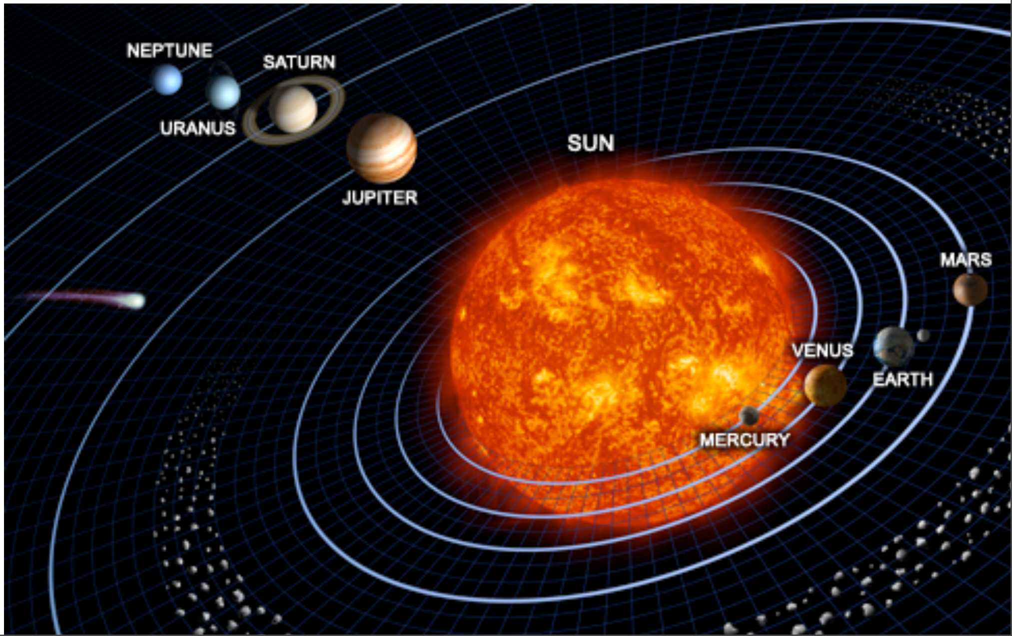
- No evolution $[H, \rho] = 0$
- Infinite temperature equilibrium
- No second law of thermodynamics
- No classical behavior. $< \phi^2(R) >= \infty$

All inconsistent with observation.

A theory of the
quantum state
of the universe
is as much a part of a
final theory
as a theory of dynamics.

Cosmology -- An Environmental Science

- What regularities of the universe can mainly be attributed to the dynamics H and what mainly to $|\Psi\rangle$?
- Roughly, regularities in time are due to H and regularities in space are to $|\Psi\rangle$.
- Solar system: Periods due to H, existence of the solar system and many more like it owes much to $|\Psi\rangle$.



ADM Quantum Cosmology

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Quantum Cosmology. I*†

CHARLES W. MISNER

The methods which ADM developed with the aim of quantizing Einstein's theory of gravity can be applied … to models of the Universe … Our main interest is directed toward quantum effects on the singularity at the beginning of time …

quantum properties of the electromagnetic field by solving the Schrödinger equation for a harmonic oscillator. For the gravitational case where we wish to retain nonlinear effects near the cosmological singularity, Fourier analysis is not the appropriate tool, but something similar is achieved by imposing a definite space

the modes which have been neglected are those at high wave number. But precisely in the limit of high wave number, good approximations[7–9] are available in the classical theory which show (a) that different modes of high wave number do not interact with each other in the

40 Years Later....

- A wave function of the universe.

But now as part of a final theory.

- Minisuperspace models

But perhaps more motivated by particle physics.

- The question of the validity of the classical Einstein equation in a quantum world.

The subject of today's talk!

The Quasiclassical Realm - A feature of our Quantum Universe

The wide range of time, place and scale on which the deterministic laws of classical physics hold to an excellent approximation.

- Time --- from the Planck era forward.
- Place --- everywhere in the visible universe.
- Scale --- macroscopic to cosmological.

What is the origin of this quasiclassical realm in a quantum universe characterized fundamentally by indeterminacy and distributed probabilities?

What is the origin of classical certainty in a quantum world?

Ehrenfest Deriv. of Classical Eqns

A particle of mass m moving in one dimension x.

Ehrenfest's Theorem: $$m\frac{d^2\langle x\rangle}{dt^2} = -\left\langle \frac{dV}{dx} \right\rangle$$

For special states, typically narrow wave packets this becomes an equation of motion for the expected value:

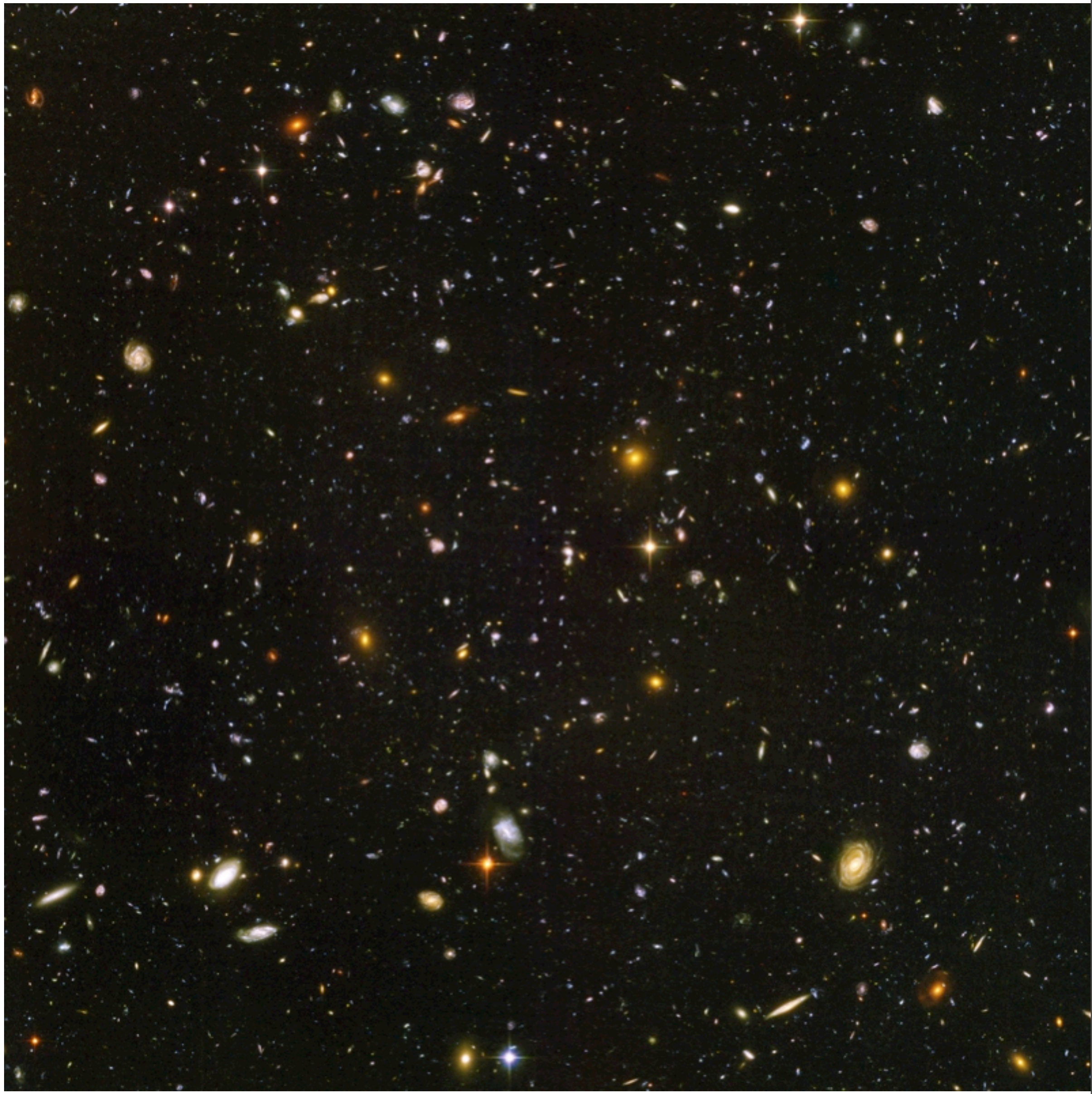
$$m\frac{d^2\langle x\rangle}{dt^2} \approx -\frac{dV(\langle x(t)\rangle)}{dx}$$

If a series of measurements is made with sufficient imprecision not to disturb this approximation the expected value will follow Newton's law.

We are not just interested in classical behavior of a few measured degrees of freedom.

We are interested in classical behavior over the whole of the visible universe over most of its history.

So the quasiclassical realm is a feature of the universe, and not our choice.



Necessary Requirements for Classical Behavior

As seen in the Ehrenfest derivation.

Coarse graining of a particular kind.

Some restriction on the state.

Classical Spacetime is the key to the origin of the rest of the quasiclassical realm.

Origin of the Quasiclassical Realm

- Classical spacetime emerges from the quantum gravitational fog at the beginning.
- Local Lorentz symmetries imply conservation laws.
- Sets of histories defined by averages of densities of conserved quantities over suitably small volumes decohere.
- Approximate conservation implies these quasiclassical variables are predictable despite the noise from decoherence.
- Local equilibrium implies closed sets of equations of motion governing classical correlations in time.

Only Certain States Lead to Classical Predictions

- Classical orbits are not predictions of every state in the quantum mechanics of a particle.
- Classical spacetime is not a prediction of every state in quantum gravity.

Classical Spacetime is the Key to the Origin of the Quasiclassical Realm.

The quantum state of the universe
is the key to the origin of
classical spacetime
in a quantum theory of gravity

A simple, discoverable theory of the universe's quantum state will not predict a unique classical history but rather the probabilities of an ensemble of possible classical histories.

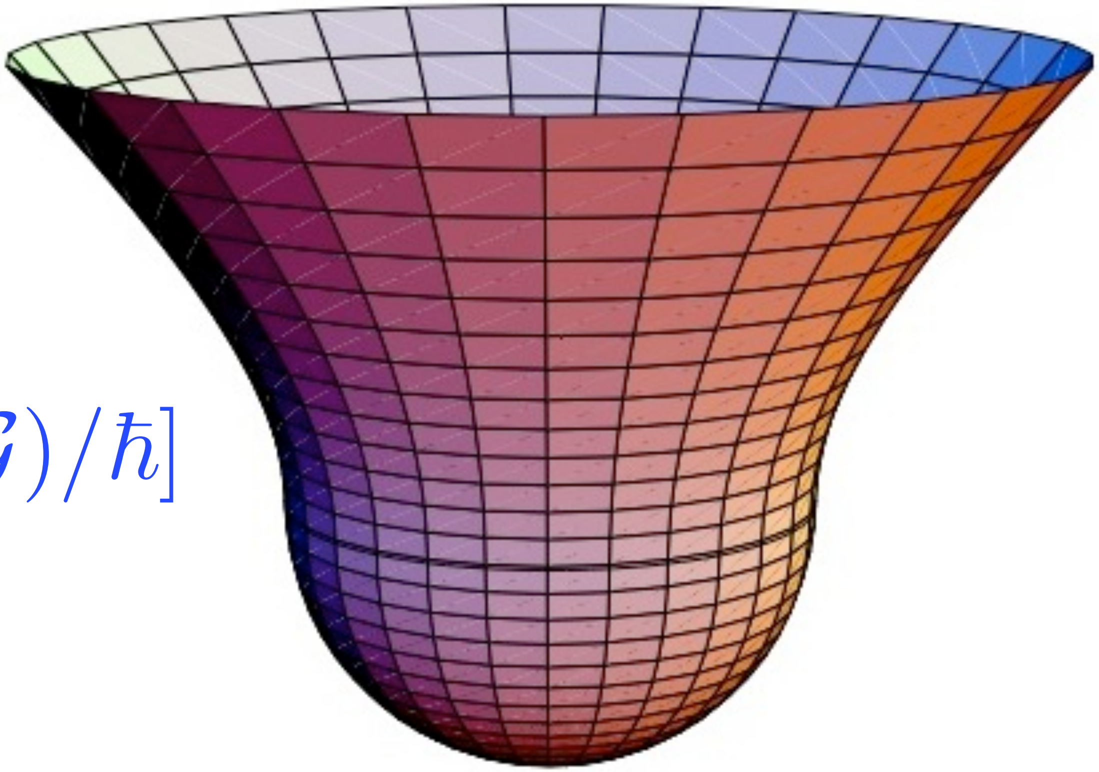
What classical spacetimes are allowed
is a central question
in quantum cosmology.

The Classical Spacetimes of Our Universe

We seek a state that will not just predict some classical spacetime but which predicts classical spacetimes with a high probability for properties consistent with our cosmological observations.

- homogeneity and isotropy
- the amount of matter
- the amount of inflation
- a spectrum of density fluctuations consistent with the CMB and growth of large scale structure
- The thermodynamic arrow of time.

The No-Boundary Quantum State of the Universe



$$\Psi(^3\mathcal{G}) = \sum_{^4\mathcal{G}} \exp[-I(^4\mathcal{G})/\hbar]$$

We analyze the ensemble of classical spacetimes predicted by Hawking's no-boundary quantum state in a simple model:

- **Low Energy Quantum Gravity** based on Einstein action:

$$I_C[g] = -\frac{m_p^2}{16\pi}\int_M d^4x(g)^{1/2}(R-2\Lambda) + (\text{surface terms})$$

- **Geometry:** homogeneous, isotropic, spatially closed:

$$ds^2 = (3/\Lambda)\left[N^2(\lambda)d\lambda^2 + a^2(\lambda)d\Omega_3^2\right]$$

- **Matter:** cosmological constant Λ plus homogeneous scalar field Φ moving in a quadratic potential.

$$V(\Phi) = \frac{1}{2}m^2\Phi^2$$

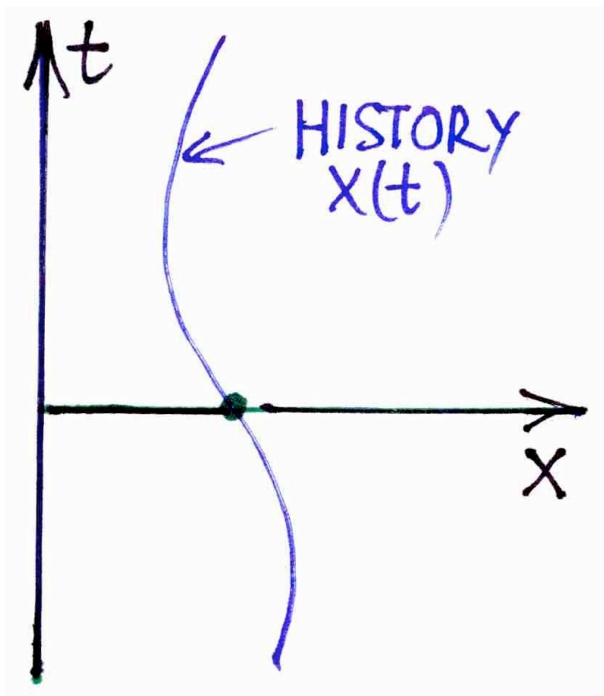
Wave Functions for the Universe

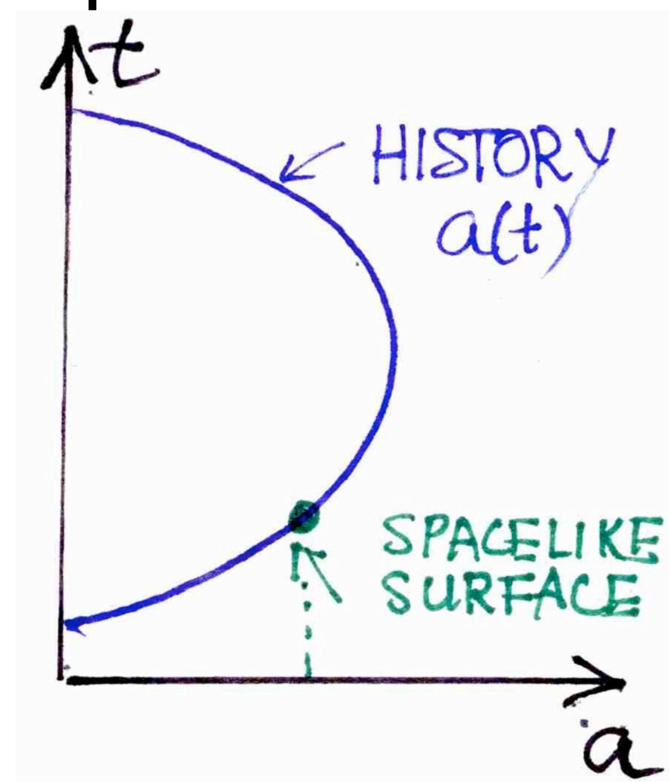
(minisuperspace models)

Geometry: Homogeneous, isotropic, closed.

$$ds^2 = (3/\Lambda)\left[N^2(\lambda)d\lambda^2 + a^2(\lambda)d\Omega_3^2\right]$$

Matter: cosmological constant plus scalar field





$$\psi = \psi(x, t)$$

$$\Psi = \Psi(b, \chi)$$

Ground State of SHO

Two methods

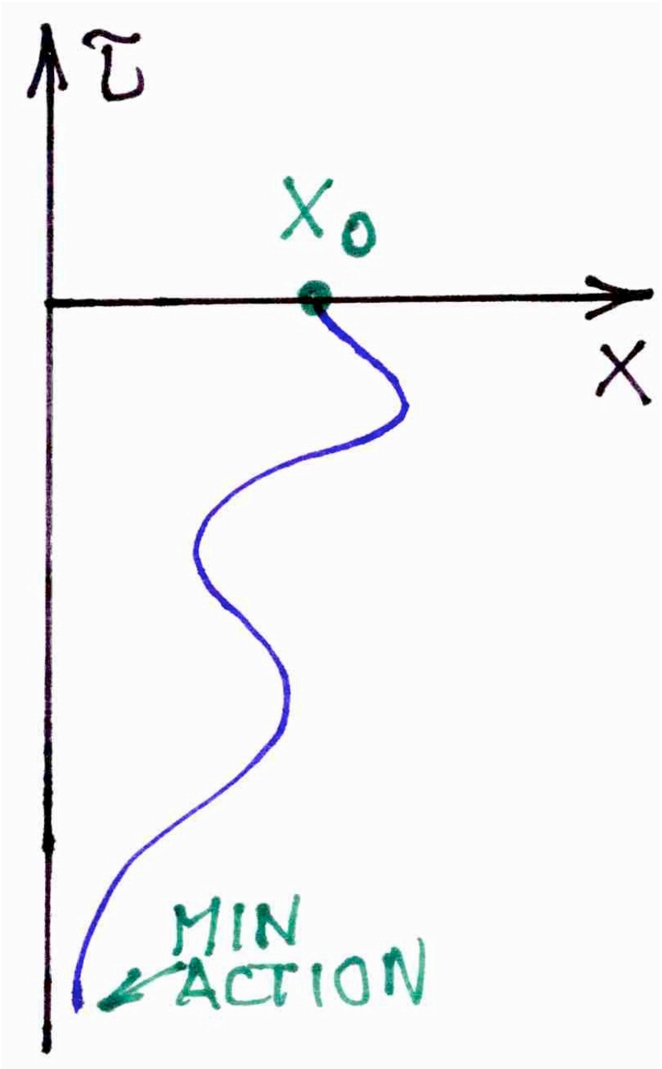
Lowest Eigenvalue of the Hamiltonian $H\psi = E\psi$.

Euclidean sum-over histories:

$$\psi(x_0) = \int \delta x \exp\{-I[x(\tau)]/\hbar\}$$

$$I[x(\tau)] = \frac{1}{2}\int d\tau[\dot{x}^2 + \omega^2 x^2]$$

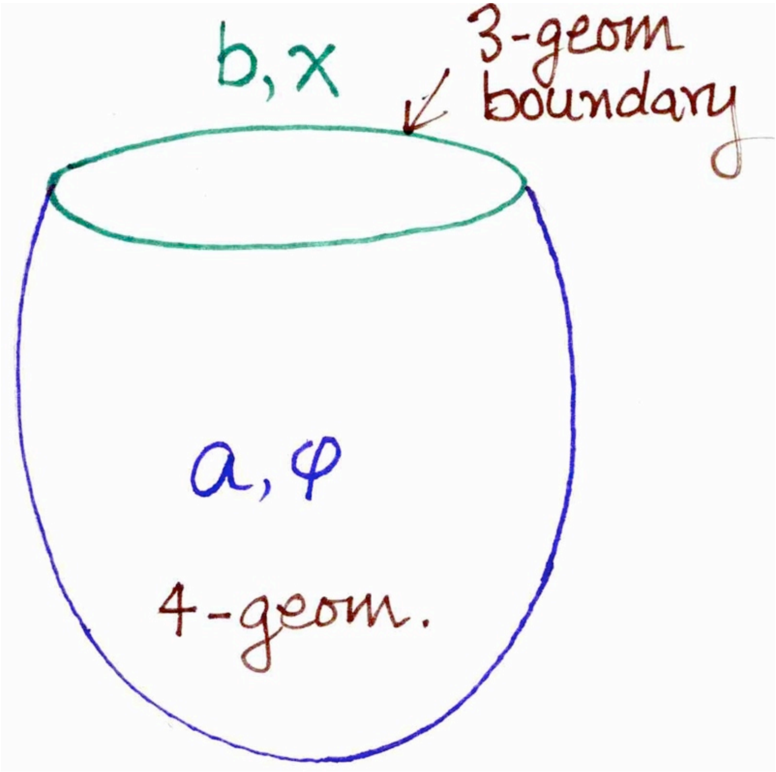
$$\psi(x_0) \propto \exp(-\omega x_0^2/2)$$



Hawking's No-Boundary Wave Function

Cosmological analog of ground state

~~No H to be a lowest eigenvalue of,~~
~~for a closed universe $H = 0$~~



Euclidean sum over all four geometries with one boundary for the arguments of the wave function and no other.

$$\Psi(b,\chi) \equiv \int_C \delta N \delta a \delta\phi \exp(-I[N(\lambda), a(\lambda), \phi(\lambda)]/\hbar) \quad .$$

Classical spacetime is predicted in states for which the probability is high for decoherent histories exhibiting patterns of correlation implied of the Einstein equation.

Classical Pred. in NRQM ---Key Points

Semiclassical form:

$$\Psi(q_0) = A(q_0)e^{iS(q_0)/\hbar}$$

- When $S(q_0)$ varies rapidly and $A(q_0)$ varies slowly, high probabilities are predicted for classical correlations in time of suitably coarse grained histories.
- For each q_0 there is a classical history with probability:

$$p_0 = \nabla S(q_0) \qquad p(\text{class.hist.}) = |A(q_0)|^2$$

Semiclassical Approx. for the NBWF

$$\Psi(b,\chi) \equiv \int_C \delta N \delta a \delta\phi \exp(-I[N(\lambda), a(\lambda), \phi(\lambda)]/\hbar)$$

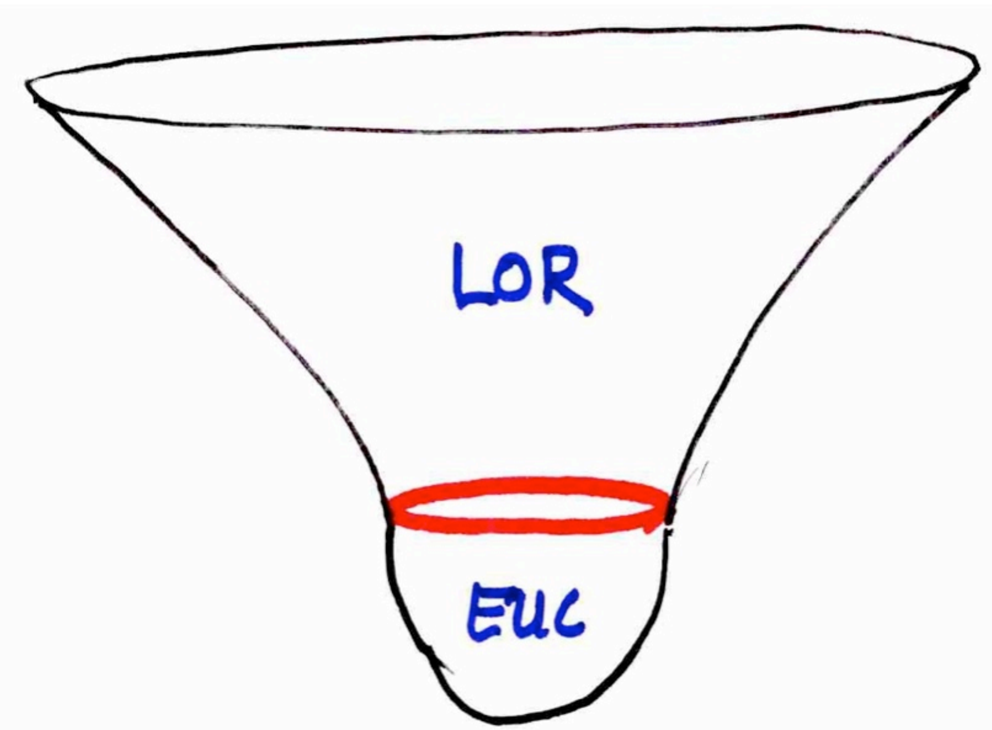
- In certain regions of superspace the steepest descents approximation may be ok.
- To leading order in ħ the NBWF will then have the semiclassical form:

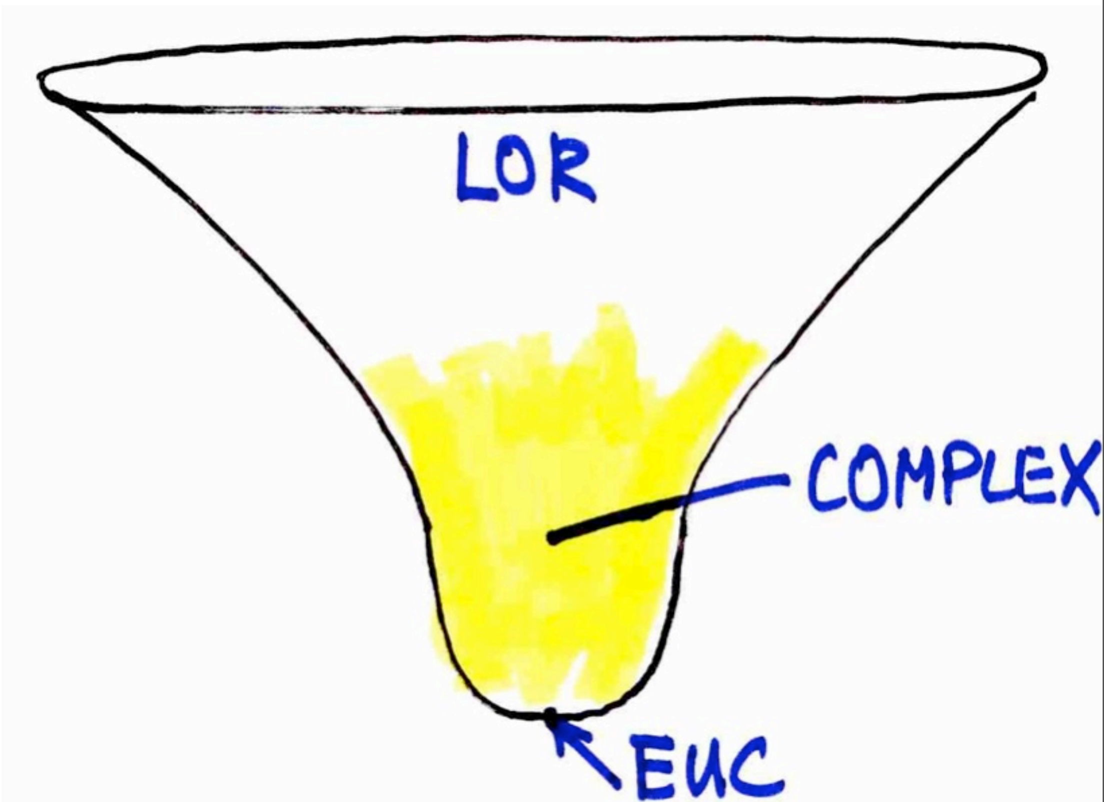
$$\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\}.$$

- The next order will contribute a prefactor which we neglect. Our probabilities are therefore only relative.

Instantons and Fuzzy Instantons

In simple cases the extremal geometries may be real and involve Euclidean instantons, but in general they will be a complex --- fuzzy instantons.





Classical Prediction in MSS and The Classicality Constraint

$$\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\}$$

- Following the NRQM analogy this semiclassical form will predict classical Lorentian histories that are the integral curves of S, ie the solutions to:

$$p_A = \nabla_A S \qquad\qquad p(\text{class. hist.}) \propto \exp(-2I_R/\hbar)$$

- However, we can expect this only when S is rapidly varying and I_R is slowly varying, e.g.

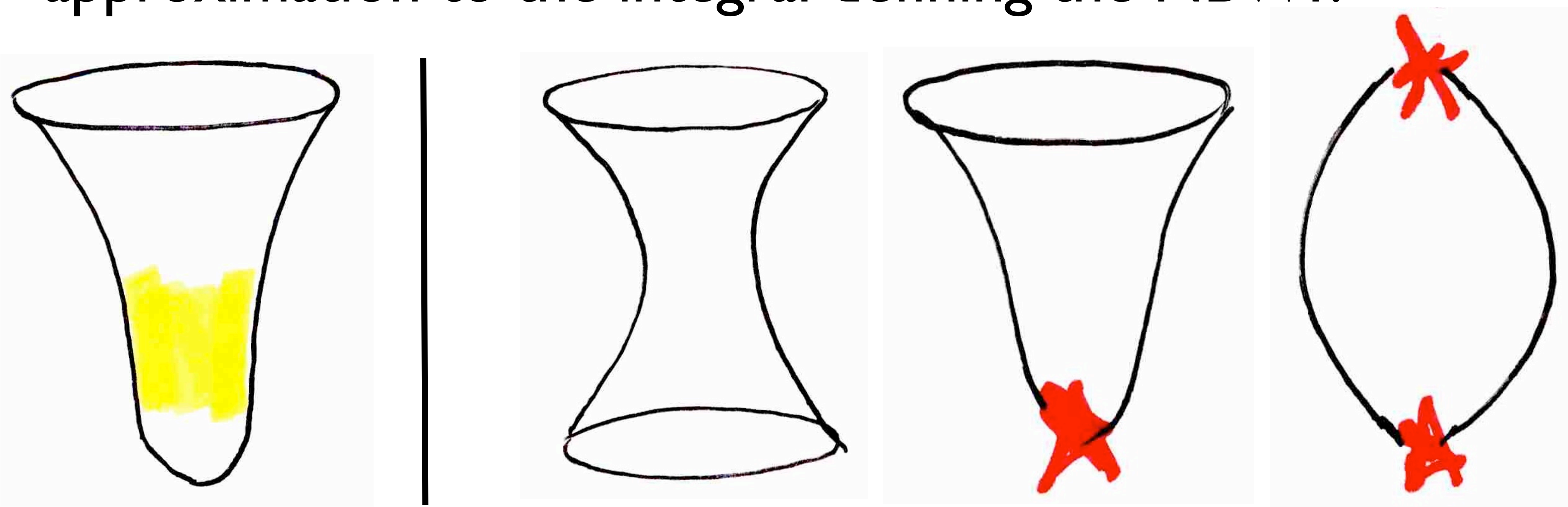
$$(\nabla I_R)^2 \ll (\nabla S)^2 .$$

This is the classicality constraint.

Class. Prediction --- Key Points

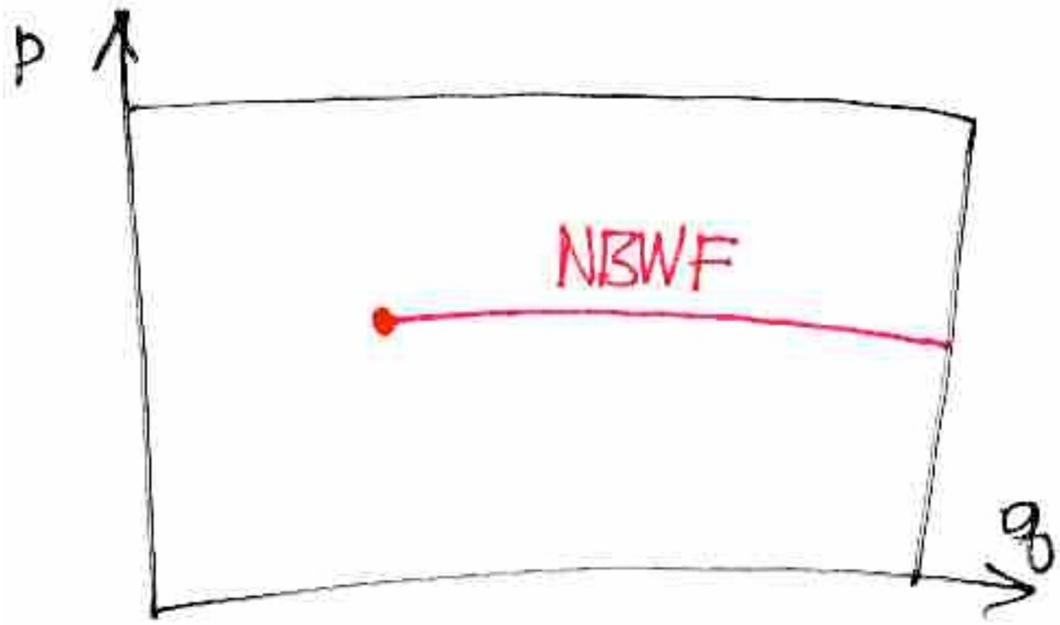
- The NBWF predicts an ensemble of entire, 4d, classical histories.

- These real, Lorentzian, histories are not the same as the complex extrema that supply the semiclassical approximation to the integral defining the NBWF.



No-Boundary Measure on Classical Phase Space

The NBWF predicts an ensemble of classical histories that can be labeled by points in classical phase space. The NBWF gives a measure on classical phase space.



The NBWF predicts a one-parameter subset of the two-parameter family of classical histories, and the classicality constraint gives that subset a boundary.

Equations and BC

$$\hbar = c = G = 1, \;\; \mu \equiv (3/\Lambda)^{1/2} m, \;\; \phi \equiv (4\pi/3)^{1/2}\Phi, \;\; H^2 \equiv \Lambda/3$$

You won't follow this.
I just wanted to show how
much work we did.

Extr

Equa

$$\ddot{a} + 2a\dot{\phi}^2 + a(1 + \mu^2\phi^2) = 0 \; .$$

Reg

Sou

The only important point is that there is one classical history for each value of the field at the south pole $\phi_0 \equiv |\phi(0)|$.

Para

matching:

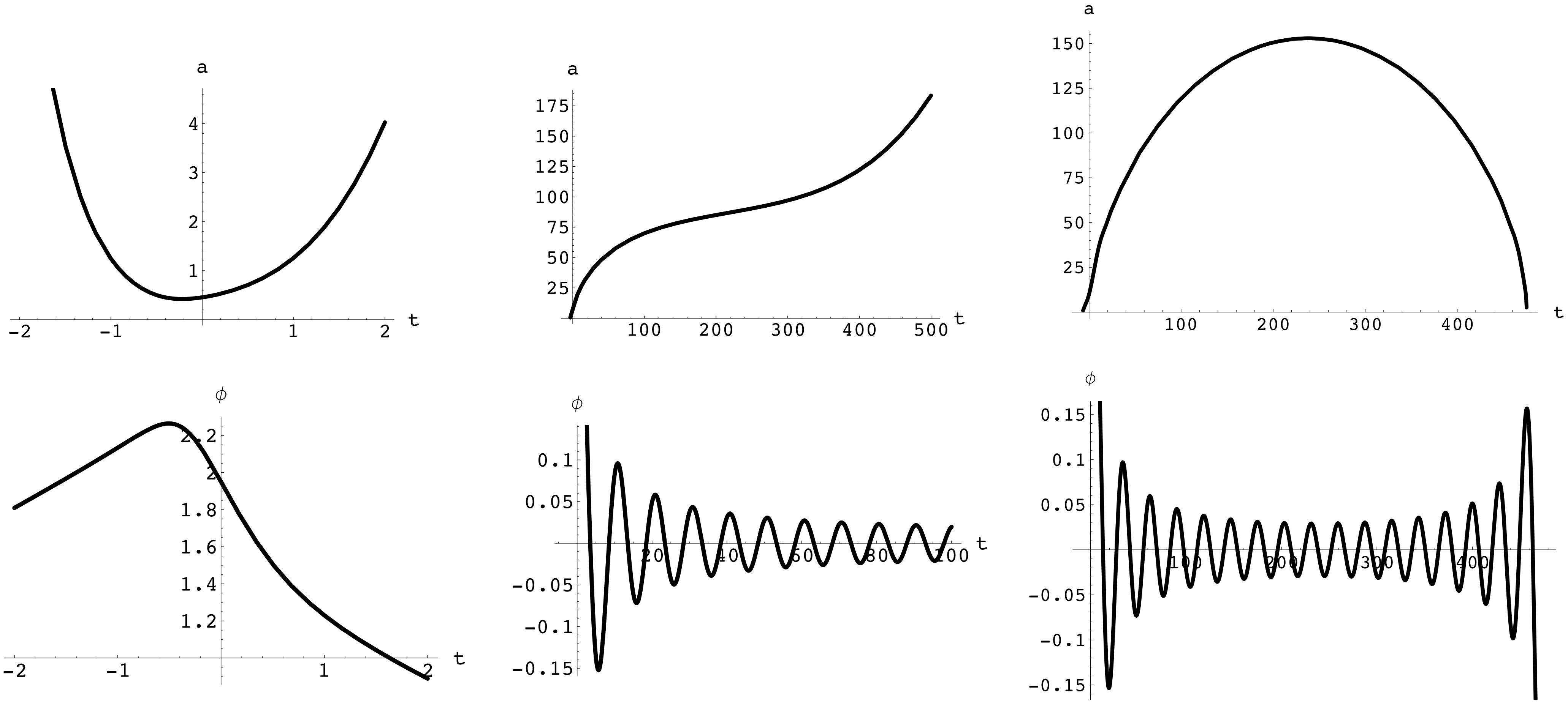
$$(\phi_0, \gamma, X, Y) \longleftrightarrow (b, \chi, 0, 0)$$

Finding Solutions

- For each ϕ_0 tune remaining parameters to find curves in (b, χ) for which I_R approaches a constant at large b.
- Those are the Lorentzian histories.
- Extrapolate backwards using the Lorentizan equations to find their behavior at earlier times.
- The result is a one-parameter family of classical histories whose probabilities are

$$p(\phi_0) \propto \exp(-2I_R)$$

Members of the Classical Ensemble

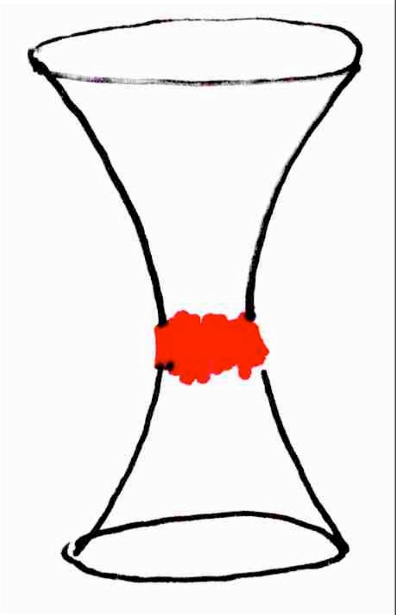


$\mu = 1.65$ $\mu = 2.25$ $\mu = 63$

$\mu \sim m/\Lambda^{1/2}$ $\phi_0 = 1.32$

Singularity Resolution

- The NBWF predicts probabilities for entire classical histories not their initial data.
- The NBWF therefore predicts probabilities for late time observables like CMB fluctuations whether or not the origin of the classical history is singular.
- The existence of singularities in the extrapolation of some classical approximation in quantum mechanics is not an obstacle to prediction by merely a limitation on the validity of the approximation.



Cosmological Questions

In the classical ensemble what is the probability that:

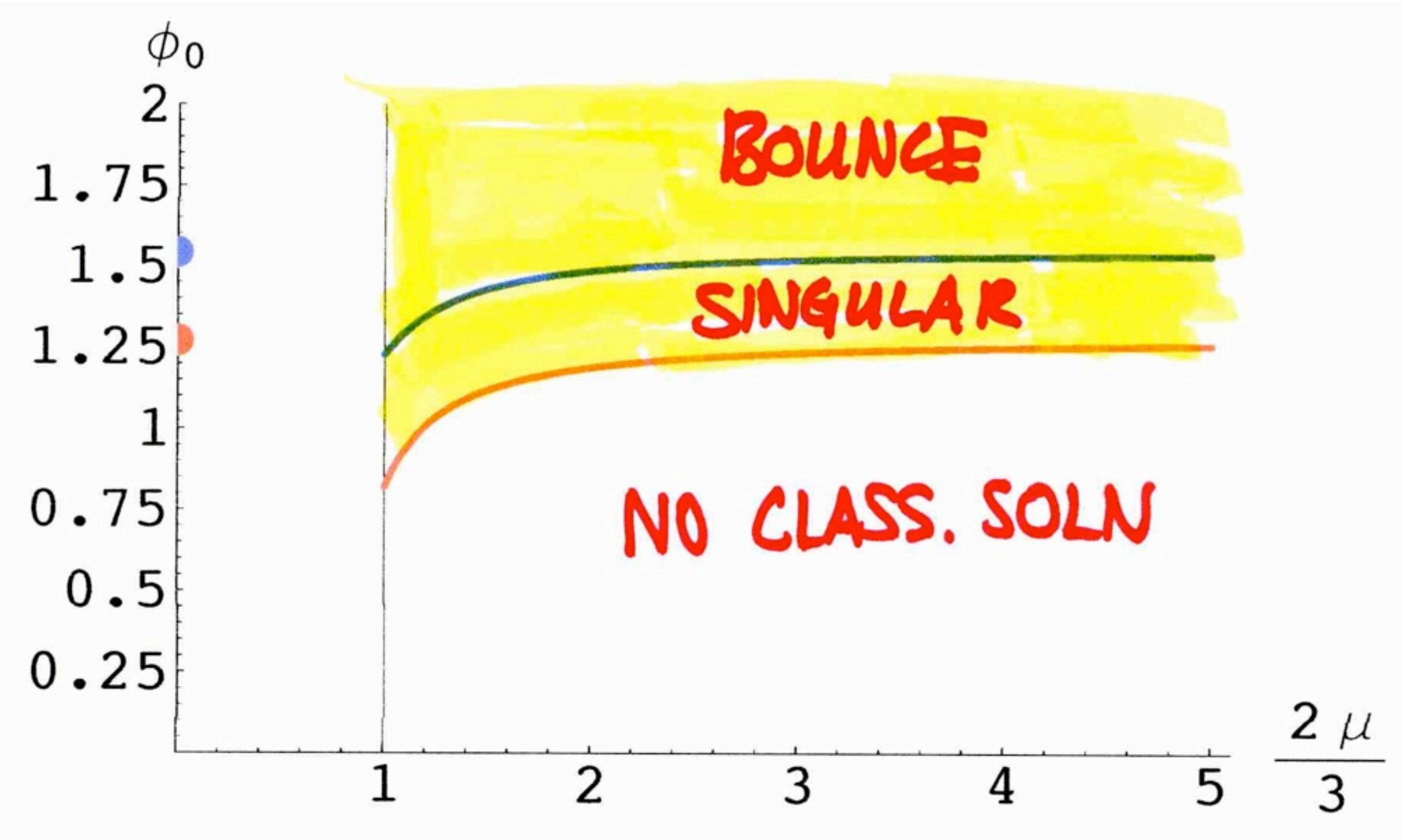
- That the universe was singular in the beginning or bounced at a small radius.
- That the universe recollapses in the future or expands forever.
- For the number of efoldings of inflation.
- For the direction of the arrow of time.
- For small fluctuations away from homogeneity and isotropy.

Answers

$$\mu = 3m/\Lambda > 3/2$$

In terms of bottom-up probabilities
conditioned only on the NBWF
for histories in the classical ensemble.

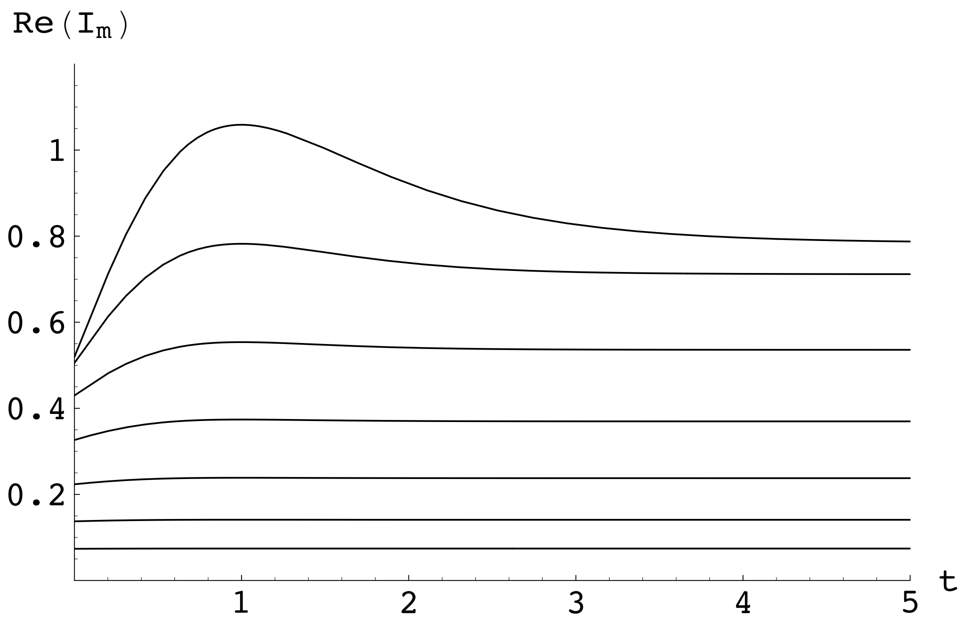
Origins



No large, nearly empty, classical universes.

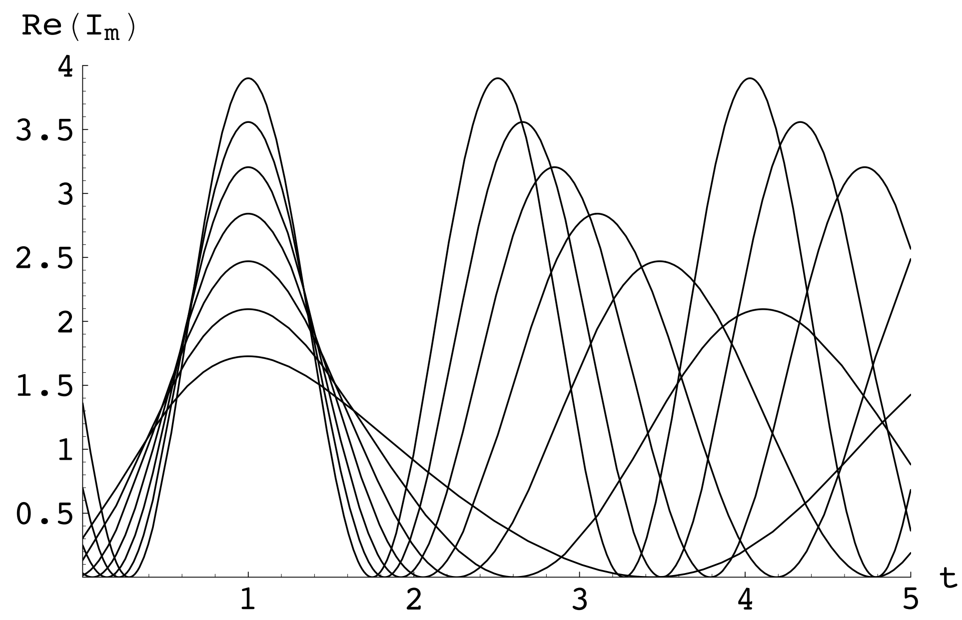
Classicality Constraint ---Pert. Th.

Small field perts on deSitter space.



$\mu < 3/2$

Classical

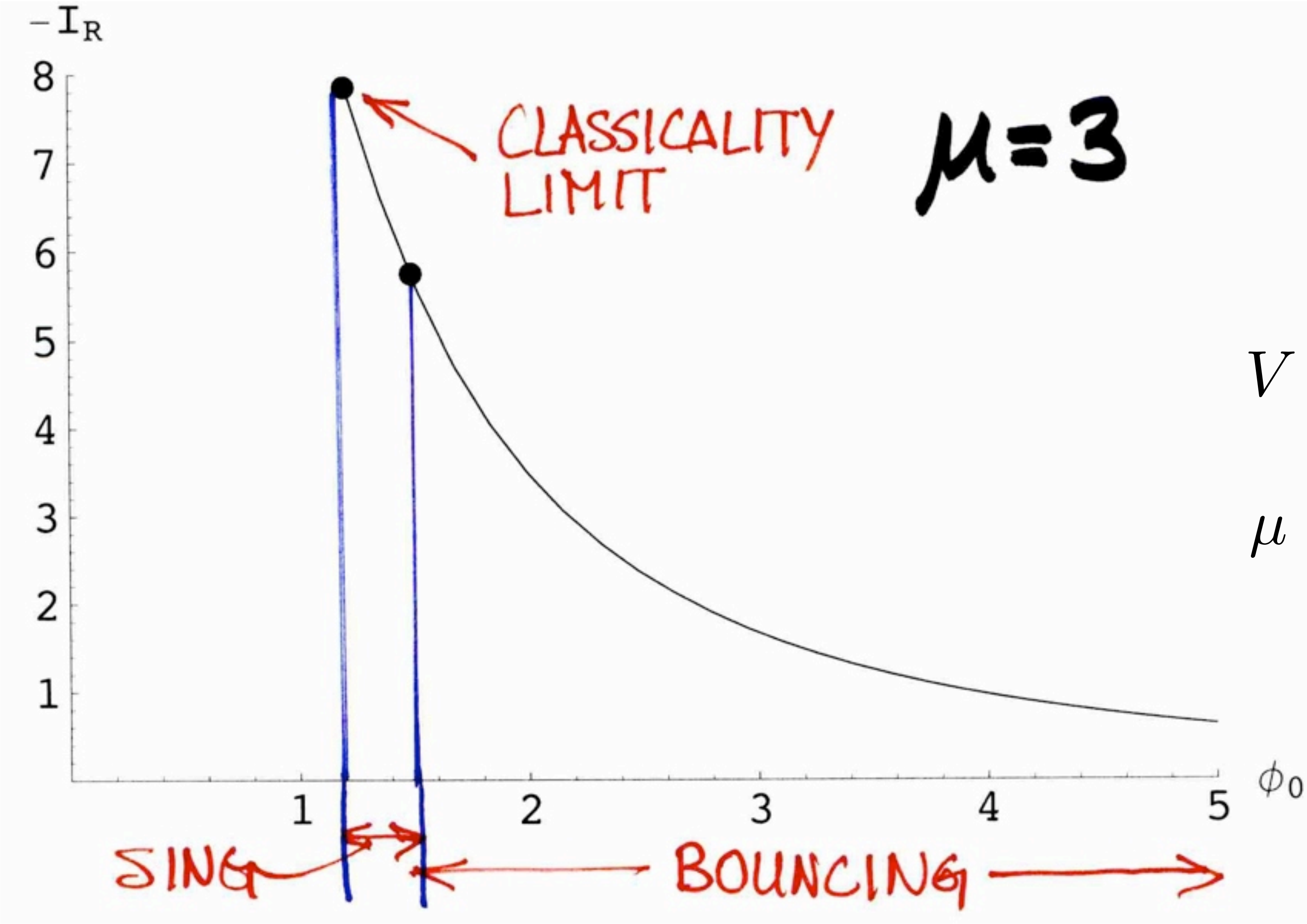


$\mu > 3/2$

Not-classical

This is a simple consequence of two decaying modes for $\mu < 3/2$, and two oscillatory modes for $\mu > 3/2$.

Probabilities and Origins



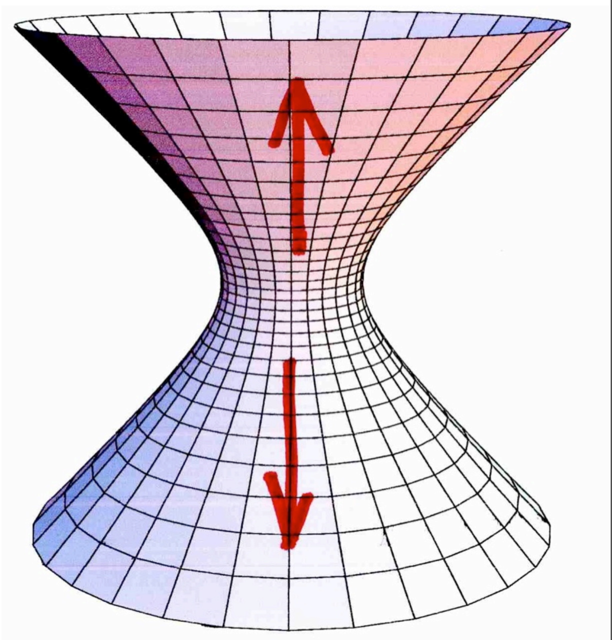
$$V = \frac{1}{2} m^2 \Phi^2$$

$$\mu \equiv (3/\Lambda) m$$

There is a significant probability that the universe never reached the Planck scale in its entire evolution.

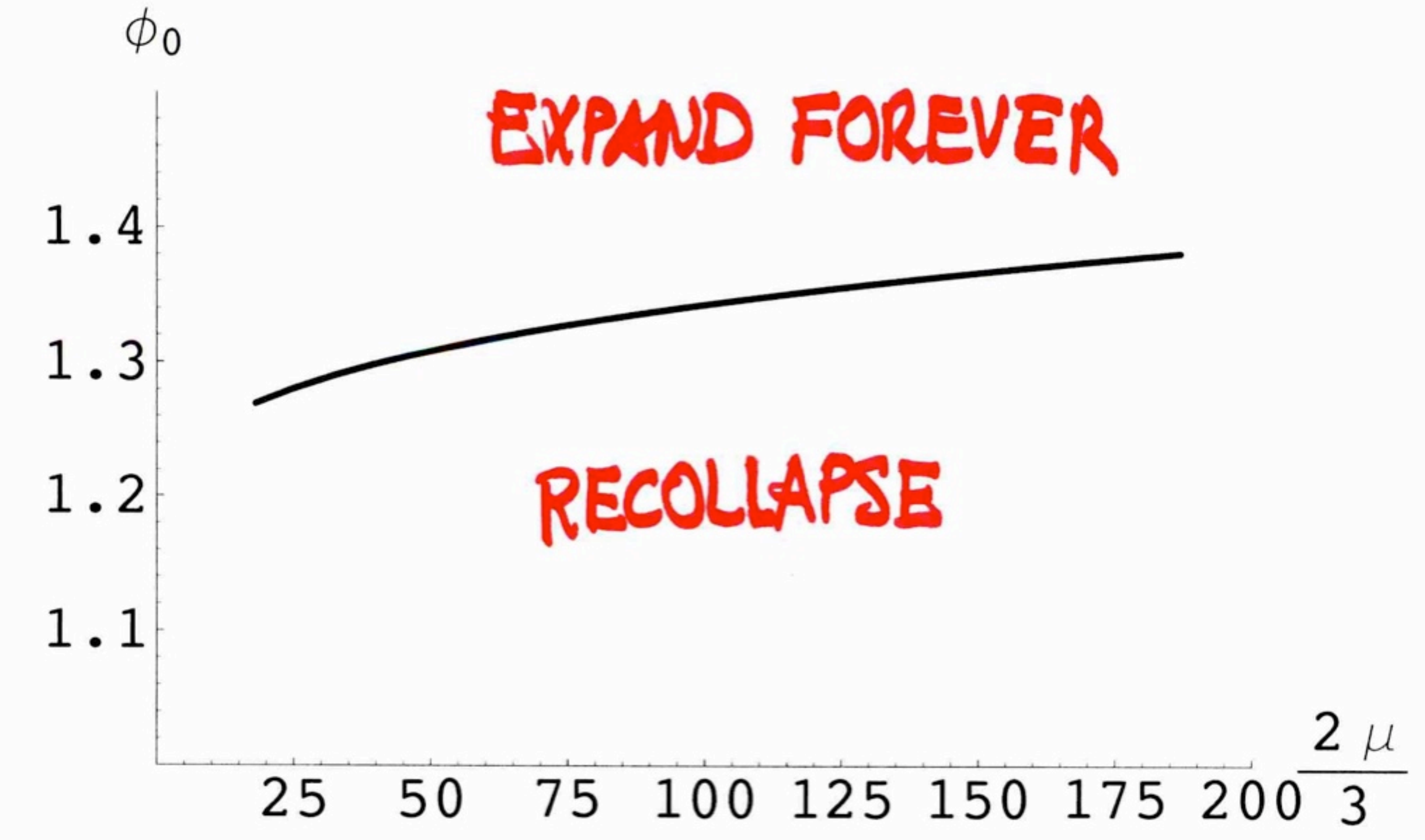
Arrows of Time

- The growth of fluctuations defines an arrow of time, order into disorder.
- NBWF fluctuations vanish at the South Pole of the fuzzy instanton.
- Fluctuations therefore increase away from the bounce on both sides.
- Time's arrow points in opposite directions on the opposite sides of the bounce.
- Events on one side will therefore have little effect on events on the other.



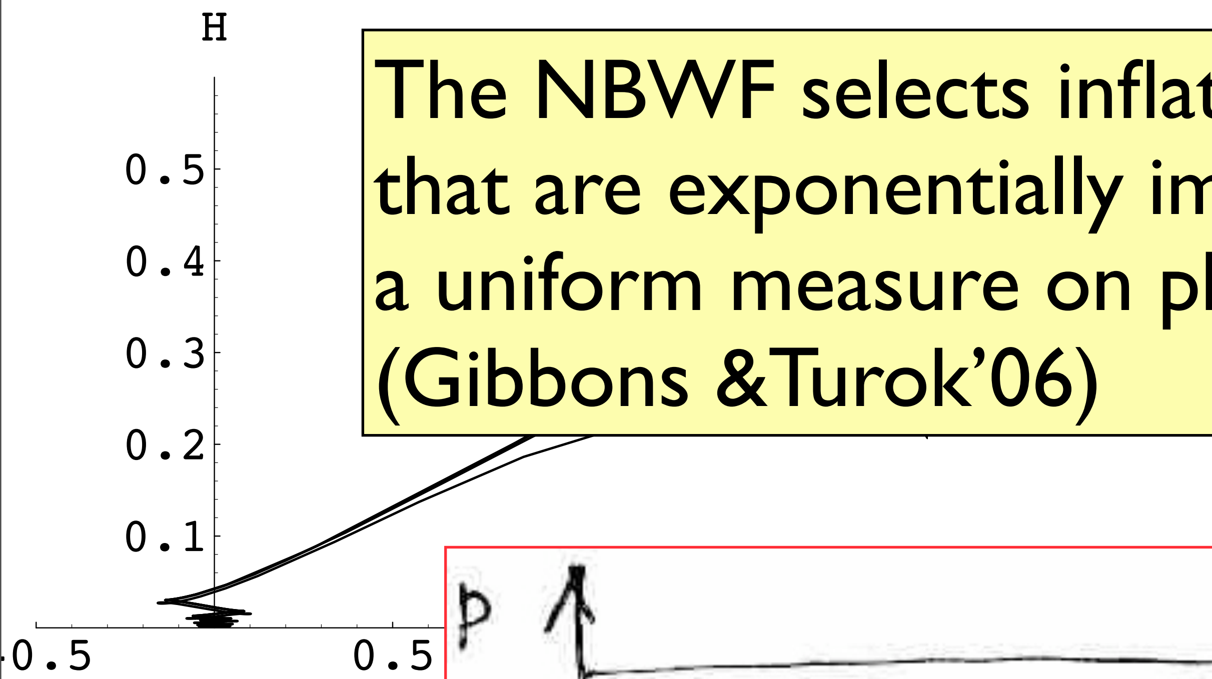
compare Caroll & Chen

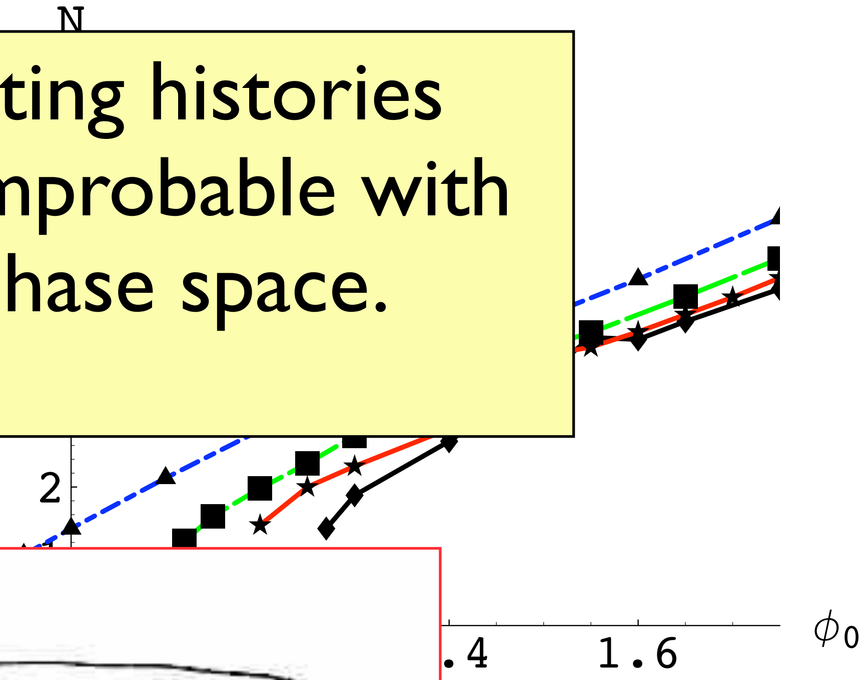
Future



Recollapse only for $\mu \sim m/\Lambda^{1/2} > 23$.

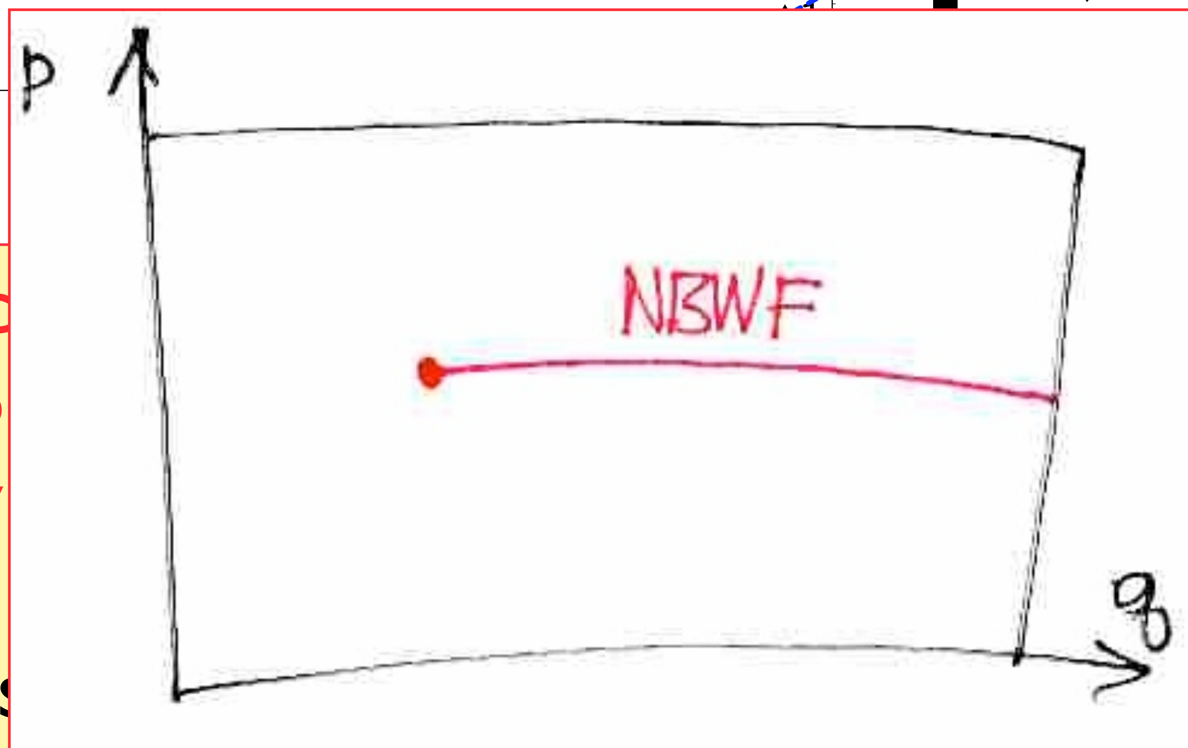
Inflation

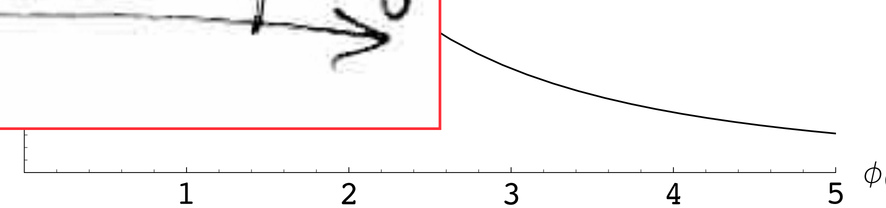




The NBWF selects inflating histories that are exponentially improbable with a uniform measure on phase space. (Gibbons &Turok'06)

There is sc
inflation fo
allowed by
constraint,
of efolding
probable of them.





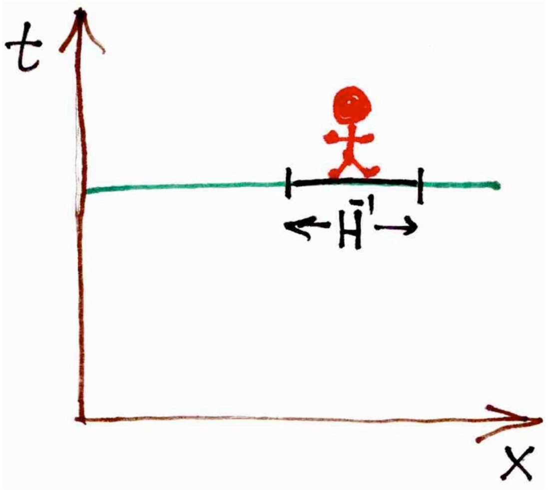
Probabilities for Our Observations

- The NBWF predicts probabilities for entire 4-d histories.
- We so not somehow observe 4-d histories from the outside.
- Rather, we are physical systems within the universe, living at some particular location in spacetime that is partially specified by our data D.
- Probabilities for observations are therefore conditioned on D.
- The probabilities for observations of the CMB for instance depend on when and where they are made.

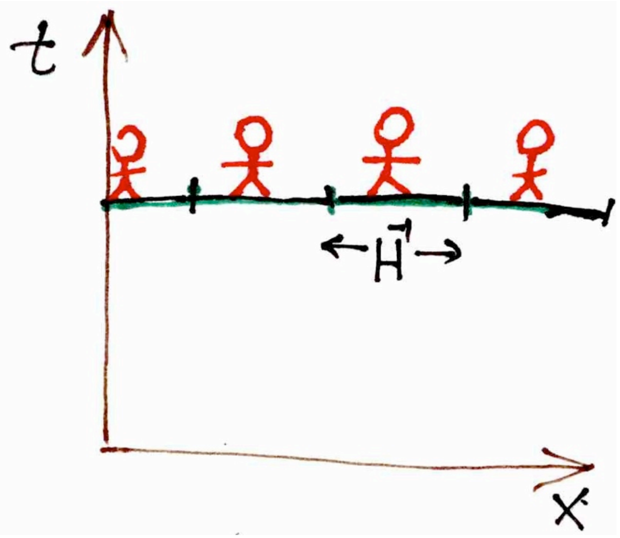
Probabilities for Observations

- The NBWF predicts probabilities for entire classical histories. (Bottom-up probabilities.)
- Our observations are restricted to a part of a light cone extending over a Hubble volume and located somewhere in spacetime.
- To get the probabilities for our observations (top-down probabilities) we sum over the bottom-up probabilities for the classical spacetimes that contain our data at least once, and then sum over the possible locations of our light cone in them.

Sum over location in homo/iso models



- Assume our data locate us on a surface of homogeneity, and approx. data on the past light cone by data in a Hubble vol. on that surface



- Assume we are rare. (If we are everywhere there is no sum).
- The sum multiplies the probability for each history ϕ_0 by

$$V_{\rm surf}/V_{\rm Hubble} \approx \exp(3N)$$

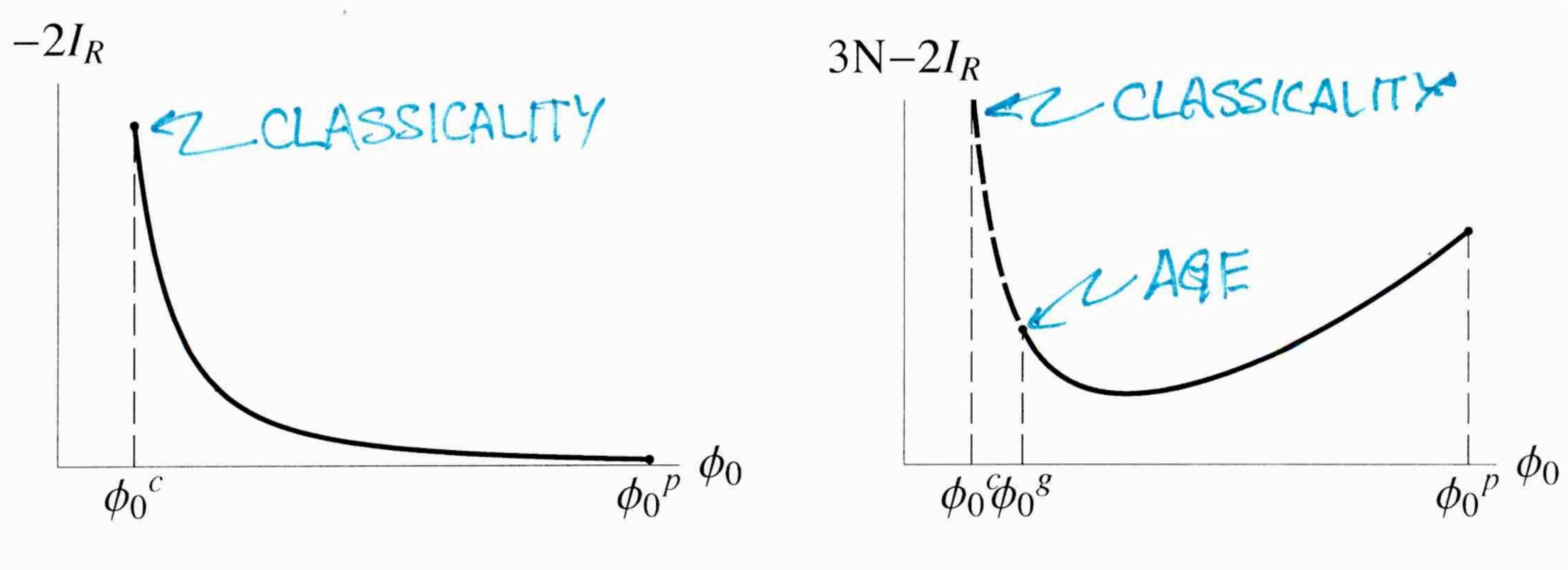
N= # efoldings

Page 97, Hawking 07

Volume Weighting favors Inflation

By itself, the NBWF + classicality favor low inflation, but we are are more likely to live in a universe that has undergone more inflation, because there are more places for us to be.

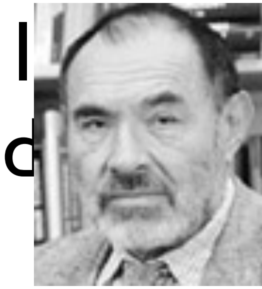
$$p(\phi_0|H_0,\rho) \propto \exp(3N)p(\phi_0) \propto \exp(3N - 2I_R)$$



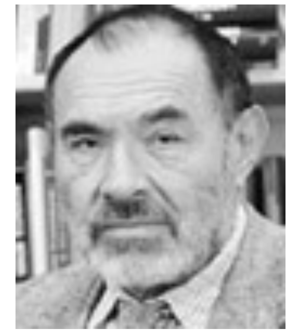
Observers are Quantum Systems within the Universe

- Volume weighting will break down in the very large (or infinite) universes contemplated by contemporary inflationary theories.
- It is then essential to take into account that observers are quantum physical subsystems within the universe that arose from quantum processes that occurred over the course of its history.

Our Model Observer

cterized by I  ta D in its past light cone
including its own d tion.

A probability $p_E(D)$ for D to exist at any one location.



$p_E(D)$

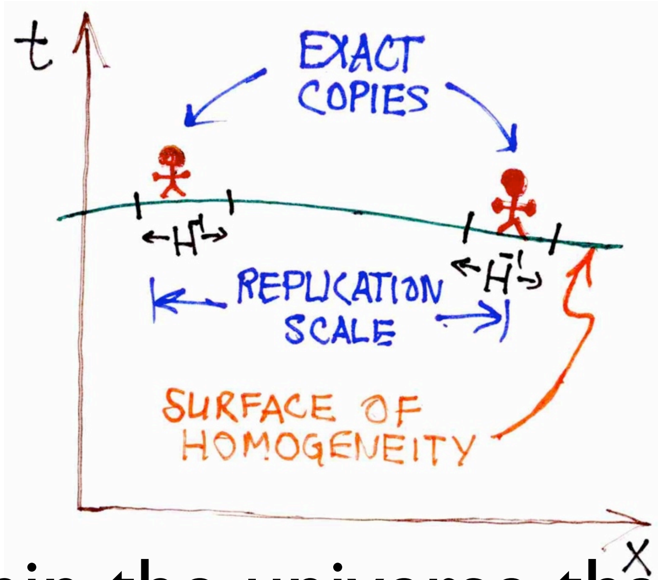
$1 - p_E(D)$

The probability p_E is very small, but in a very large
u  e the probability becomes significant that D 
w eplicated exactly elsewhere.

All we know for certain about the universe is that is has at least one copy of our data, $D^{\geq}$.

Replication and Regulation

- In an infinite universe volume weighting breaks down.
- In an infinite universe the probability is unity that we are replicated elsewhere. We are then not rare.



- We are quantum physical systems within the universe that have a probability p_E to exist in any Hubble volume.
- Rather than volume, probabilities should be weighted by the probability that the is at least one instance of us in the universe (all we know for certain).

$$1 - (1 - p_E)^{N_h}$$

Srednicki a.o. 07
Hertog a.o 09

- This is finite for infinite number of Hubble volumes N_h but reduces to volume weighting when p_E is small (rare).

Forthcoming Results on Inhomogeneous Fluctuations

- We calculated the (top-down) NBWF probabilities for small fluctuations away from homogeneity and isotropy conditioned on at least one instance of our data.
- Fluctuations on observable scales (e.g. CMB) are Gaussian whether we are rare or not.
- On large scales that left the horizon in the regime of eternal inflation the universe is predicted to be significantly inhomogenous.

The Main Points Again

- The universe has a quantum state.
- Classical spacetime is the key to the classical realm.
- Only certain states in quantum gravity predict classical cosmological spacetimes.
- The NBWF predicts an ensemble of classical cosmological spacetimes with probabilities for whether they bounce or are singular, the amount of inflation, the sizes of fluctuations, and the direction of time's arrow.
- Probabilities for our observations are conditioned on a description of us and the observational situation, implying a volume weighting of probabilities of histories.
- These probabilities favor significant inflation and a universe that is inhomogeneous on very large scales.

Questions for ADM

- Fifty years ago what did you think would be the future of quantum gravity?
- Where do we stand today and what do we have left to do?
- How do you think today's efforts will appear at ADM-100?

Thank you ADM!

For starting it all off.

I hope you like where it went!