General Relativity in Higher Dimensions

ADM-50

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- There is a vast accumulation of knowledge about exact solutions in four-dimensional General Relativity. Much less is known about the possibilities in higher dimensions.

- With the advent of supergravity and string theory, it becomes important to study higher-dimensional solutions.

- Some familiar 4-dimensional uniqueness theorems no longer apply; Structure of solution space can be much enlarged in higher dimensions.

- In string theory, five dimensions is of particular interest, because of the AdS/CFT correspondence. Especially, black holes in $D = 5$ Einstein theory with a cosmological constant, and charged black holes in gauged supergravities.

- $D = 5$ is intriguing since it admits black hole solutions not only with S^3 horizon topology, but also $S^1 \times S^2$ (black rings).

- Here, we focus first on black holes in $D \geq 5$ dimensions with S^{D-2} horizon topology. We shall describe rotating black holes without, and with, a cosmological constant; the introduction of NUT parameters; the introduction of charge (in gauged supergravity); and the some further recent generalisations in $D = 5$.

- We also look at homogeneous Einstein metrics on group manifolds, and inhomogeneous metrics on spheres, showing how the Einstein equations become less and less restrictive in higher dimensions.

Review of Kerr-AdS in Four Dimensions

The Kerr-AdS solution, satisfying $R_{\mu\nu} = -3g^2\, \mathrm{g}_{\mu\nu}$, is

$$ds_4^2 = -\frac{\Delta_r}{\rho^2}(dt - \frac{a}{\Xi}\sin^2\theta d\phi)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}(a dt - \frac{r^2+a^2}{\Xi}d\phi)^2 + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta}$$

where $\Xi = 1 - g^2 a^2$ and

$$\Delta_r = (r^2+a^2)(1+g^2r^2) - 2mr\,, \quad \Delta_\theta = 1 - a^2 g^2\cos^2\theta\,, \quad \rho^2 = r^2 + a^2\cos^2\theta$$

It is characterised by mass m and rotation a parameters. Defining new coordinates by $x = r$, $y = a\cos\theta$, $\phi = a\Xi\psi$ and $t = \tau + a^2\psi$, gives

$$ds_4^2 = -\frac{X(d\tau + y^2 d\psi)^2}{x^2+y^2} + \frac{Y(d\tau - x^2 d\psi)^2}{x^2+y^2} + (x^2+y^2)\,(\frac{dx^2}{X} + \frac{dy^2}{Y})$$

(Carter-Plebanski form of the metric.)

$$X = (a^2+x^2)(1-g^2x^2) - 2mx\,, \quad Y = (a^2-y^2)(1-g^2y^2) + 2\ell y$$

Here we have added in a NUT charge ℓ as well. (Note symmetry with mass.)

There is a complete symmetry between x and y.

The outer event horizon, at the largest root of $X(x) = 0$, is topologically S^2, and geometrically a spheroid.

Rotating Black Holes in $D \ge 5$ Dimensions

The Ricci-flat rotating black holes in $D \ge 5$ were constructed by Myers and Perry, 1980. The generalisation to include a cosmological constant was obtained by Gibbons, Lü, Page and CNP, 2004.

There are independent rotations a_i in the $[(D-1)/2]$ orthogonal 2-planes:

$$\begin{aligned} ds^2 &= -W\,(1+g^2r^2)\,dt^2 + \frac{2m}{U}\Big(W\,dt - \sum_{i=1}^{N}\frac{a_i\,\mu_i^2\,d\varphi_i}{\Xi_i}\Big)^2 \\ &\quad + \sum_{i=1}^{N}\frac{r^2+a_i^2}{\Xi_i}\,\mu_i^2\,d\varphi_i^2 + \frac{U\,dr^2}{V-2m} + \sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\Xi_i}\,d\mu_i^2 \\ &\quad - \frac{g^2}{W\,(1+g^2r^2)}\Big(\sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\Xi_i}\,\mu_i\,d\mu_i\Big)^2\,, \qquad \sum_{i=1}^{N+\epsilon}\mu_i^2 = 1 \end{aligned}$$

$$\begin{aligned} W &\equiv \sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{\Xi_i}\,, \quad U \equiv r^\epsilon \sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{r^2+a_i^2}\prod_{j=1}^{N}(r^2+a_j^2)\,, \quad \sum_{i=1}^{N+\epsilon}\mu_i^2 = 1 \\ V &\equiv r^{\epsilon-2}\,(1+g^2r^2)\prod_{i=1}^{N}(r^2+a_i^2)\,, \qquad \Xi_i \equiv 1 - g^2a_i^2 \end{aligned}$$

Satisfies $R_{\mu\nu} = -(D-1)g^2\,\mathbf{g}_{\mu\nu}$ in $D = (2N+1+\epsilon)$ dimensions ($\epsilon = 0$ or $\epsilon = 1$.).

Simplification By a Jacobi Transformation

The black hole metrics in their original form are difficult and cumbersome to work with, because of the use of the set of direction cosines μ_i as coordinates, subject to the constraint $\sum_i \mu_i^2 = 1$. For example, there is no convenient vielbein basis.

A major simplification results if we solve the constraint using a Jacobi transformation (Chen, Lü and CNP):

$$\mu_i^2 = \frac{\prod_{\alpha=1}^{n-1}(a_i^2 - y_\alpha^2)}{\prod'^{n}_{k=1}(a_i^2 - a_k^2)}\,, \qquad D = 2n+1 \;\; \text{or} \;\; D = 2n$$

($\prod'$ means omit the zero factor in the product.)

An added bonus is that generalising to include NUT parameters is now immediate (as it was in $D = 4$ once the Kerr metric was written in the Carter-Plebanski form).

"These Black Holes May Contain NUTS"

$$ds^2 = \sum_{\mu=1}^{n} \Big\{ \frac{U_\mu}{X_\mu}\, dx_\mu^2 + \frac{X_\mu}{U_\mu} \Big[\frac{W}{1 - g^2 x_\mu^2}\, dt - \sum_{i=1}^{n-1} \frac{\gamma_i}{a_i^2 - x_\mu^2}\, d\phi_i \Big]^2 \Big\} \qquad D = 2n$$

where

$$U_\mu = {\prod}'^{n}_{\nu=1} (x_\nu^2 - x_\mu^2)\,, \qquad X_\mu = -(1 - g^2 x_\mu^2) \prod_{k=1}^{n-1} (a_k^2 - x_\mu^2) - 2M_\mu\, x_\mu$$

$$W = \prod_{\nu=1}^{n} (1 - g^2 x_\nu^2)\,, \qquad \gamma_i = \prod_{\nu=1}^{n} (a_i^2 - x_\nu^2)$$

$$\begin{aligned} ds^2 \;=\; & \sum_{\mu=1}^{n} \Big\{ \frac{U_\mu}{X_\mu}\, dx_\mu^2 + \frac{X_\mu}{U_\mu} \Big[\frac{W}{1 - g^2 x_\mu^2}\, dt - \sum_{i=1}^{n} \frac{a_i^2\, \gamma_i}{a_i^2 - x_\mu^2}\, d\phi_i \Big]^2 \Big\} \\ & - \frac{\prod_{k=1}^{n} a_k^2}{\prod_{\mu=1}^{n} x_\mu^2} \Big[W\, dt - \sum_{i=1}^{n} \gamma_i\, d\phi_i \Big]^2 \end{aligned} \qquad D = 2n + 1$$

with

$$X_\mu = \frac{(1 - g^2 x_\mu^2)}{x_\mu^2} \prod_{k=1}^{n} (a_k^2 - x_\mu^2) + 2M_\mu$$

M_n is the mass parameter. M_a, for $1 \le \alpha \le n - 1$, are NUT parameters.

Example: Five-Dimensional Kerr-NUT-AdS

In five dimensions, after a further coordinate transformation, the Kerr-NUT-AdS metric can be written as

$$
\begin{aligned}
ds_5^2 \;=\;& (x+y)\Big(\frac{dx^2}{4X}+\frac{dy^2}{4Y}\Big) - \frac{X}{x(x+y)}(dt+y\,d\phi)^2 + \frac{Y}{y(x+y)}(dt-x\,d\phi)^2 \\
&+\frac{a^2b^2}{xy}\Big(dt - xy\,d\chi - (x-y)d\phi\Big)^2
\end{aligned}
$$

where

$$X=(x+a^2)(x+b^2)(1+g^2x)-2m\,x\,,\quad Y=-(a^2-y)(b^2-y)(1-g^2y)-2\ell\,y$$

The generalisation to $D=2n+1\ge 7$ follows the same pattern.

The presence of the NUT parameter ℓ here is actually illusory. In $D=2n+1$ dimensions, although one can ostensibly have $n-1$ NUT parameters, one of them can always be removed by coordinate transformations.

Example: Six-Dimensional Kerr-NUT-AdS

In six dimensions, after further linear coordinate transformations, we can write the Kerr-NUT-AdS metric as

$$
\begin{aligned}
ds_6^2 \;=\;& \frac{(x^2-y^2)(x^2-z^2)dx^2}{X} + \frac{(y^2-x^2)(y^2-z^2)dy^2}{Y} + \frac{(z^2-x^2)(z^2-y^2)dz^2}{Z} \\
& + \frac{X}{(x^2-y^2)(x^2-z^2)}[dt + (y^2+z^2)d\psi_1 + y^2 z^2 d\psi_2]^2 \\
& + \frac{Y}{(y^2-x^2)(y^2-z^2)}[dt + (x^2+z^2)d\psi_1 + x^2 z^2 d\psi_2]^2 \\
& + \frac{Z}{(z^2-x^2)(z^2-y^2)}[dt + (x^2+y^2)d\psi_1 + x^2 y^2 d\psi_2]^2
\end{aligned}
$$

where

$$
\begin{aligned}
X &= -(1-g^2x^2)(a^2-x^2)(b^2-x^2) - 2M\,x \\
Y &= -(1-g^2y^2)(a^2-y^2)(b^2-y^2) - 2L_1\,y\,, \\
Z &= -(1-g^2z^2)(a^2-z^2)(b^2-z^2) - 2L_2\,z
\end{aligned}
$$

Cohomogeneity-3 metric with rotations a and b, mass M, and two NUT charges L_1 and L_2.

The metrics in $D = 2n \geq 8$ follow a similar pattern.

Charged Rotating Black Holes in Higher Dimensions

In four dimensions, the generalisation of the Kerr solution to include electric charge is rather straightforward. No exact solutions describing charged rotating black holes solutions of the **pure** Einstein-Maxwell equations in higher dimensions have been found. $\mathcal{L} = \sqrt{-\mathrm{g}}(R - \frac{1}{4}F^2 - \Lambda)$

If one extends the Einstein-Maxwell system to the bosonic sector of an appropriate supergravity theory, then exact solutions *are* known. In fact, in the case of zero cosmological constant (i.e. ungauged supergravity), the introduction of electric charges is a purely mechanical procedure. One uses global symmetries of the theory to generate charged solutions from uncharged ones. (Extensively implemented by Cvetic and Youm.)

With a non-vanishing cosmological constant (gauged supergravity), the solution-generating technique fails, since there are no longer global symmetries. Brute force calculations have succeeded in various cases (Chong, Cvetic, Lü, Mei, CNP), including, for example, the general solution in five-dimensional minimal gauged supergravity. The relevant bosonic theory is described by

$$\mathcal{L} = \sqrt{-\mathrm{g}}\,(R - \tfrac{1}{4}F_{\mu\nu}\,F^{\mu\nu} + 12g^2) + \frac{1}{12\sqrt{3}}\varepsilon^{\mu\nu\rho\sigma\lambda}\,F_{\mu\nu}\,F_{\rho\sigma}\,A_\lambda$$

The solution has four parameters, characterising the mass, the charge and the two independent angular momenta:

$$
\begin{aligned}
ds_5^2 &= -\frac{\Delta_\theta\,[(1+g^2r^2)\rho^2 dt + 2q\nu]\,dt}{\Xi_a\,\Xi_b\,\rho^2} + \frac{2q\,\nu\omega}{\rho^2} + \frac{f}{\rho^4}\Big(\frac{\Delta_\theta\,dt}{\Xi_a\Xi_b} - \omega\Big)^2 \\
&\quad + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta} + \frac{r^2+a^2}{\Xi_a}\sin^2\theta d\phi^2 + \frac{r^2+b^2}{\Xi_b}\cos^2\theta d\psi^2\,, \\
A &= \frac{\sqrt{3}q}{\rho^2}\Big(\frac{\Delta_\theta\,dt}{\Xi_a\,\Xi_b} - \omega\Big)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\nu &= b\sin^2\theta d\phi + a\cos^2\theta d\psi\,, \\
\omega &= a\sin^2\theta\frac{d\phi}{\Xi_a} + b\cos^2\theta\frac{d\psi}{\Xi_b}\,, \\
\Delta_\theta &= 1 - a^2g^2\cos^2\theta - b^2g^2\sin^2\theta\,, \qquad \Xi_a = 1 - a^2g^2\,, \qquad \Xi_b = 1 - b^2g^2\,, \\
\Delta_r &= \frac{(r^2+a^2)(r^2+b^2)(1+g^2r^2) + q^2 + 2abq}{r^2} - 2m \\
\rho^2 &= r^2 + a^2\cos^2\theta + b^2\sin^2\theta\,, \qquad f = 2m\rho^2 - q^2 + 2abqg^2\rho^2
\end{aligned}
$$

The four conserved charges (mass, electric charge and angular momenta) are

$$
\begin{aligned}
E &= \frac{m\pi(2\Xi_a + 2\Xi_b - \Xi_a\,\Xi_b) + 2\pi qabg^2(\Xi_a + \Xi_b)}{4\Xi_a^2\,\Xi_b^2}\,, \qquad Q = \frac{\sqrt{3}\pi q}{4\Xi_a\,\Xi_b} \\
J_a &= \frac{\pi[2am + qb(1+a^2g^2)]}{4\Xi_a^2\,\Xi_b}\,, \qquad J_b = \frac{\pi[2bm + qa(1+b^2g^2)]}{4\Xi_b^2\,\Xi_a}
\end{aligned}
$$

Further Generalisation in Five Dimensions

In four dimensions, the Plebanski-Carter form of the Kerr-NUT-AdS metric admits a further generalisation, by Plebanski and Demianski, where a conformal factor is introduced:

$$ds_4^2 = \frac{1}{(1-xy)^2}\Big\{-\frac{X(d\tau+y^2d\psi)^2}{x^2+y^2}+\frac{Y(d\tau-x^2d\psi)^2}{x^2+y^2}+(x^2+y^2)(\frac{dx^2}{X}+\frac{dy^2}{Y})\Big\}$$

This is Einstein for certain polynomials $X(x)$ and $Y(y)$. Describes rotating AdS black holes with mass, NUT charge and acceleration. (General Type D.)

Since we can now write the higher-dimensional rotating AdS black holes with NUT charges in a form analogous to Carter and Plebanski, it is natural to seek the analogous further generalisation.

Introduction of an overall conformal factor no longer gives any non-trivial extensions. We find only one higher-dimensional case where a related trick is successful: $D=5$ Ricci-flat metrics (Lü, Mei and CNP):

$$\begin{aligned} ds_5^2 &= \frac{1}{(1-xy)^2}\Big[\frac{x-y}{4X}dx^2+\frac{y-x}{4Y}dy^2+\frac{X\,(d\phi+yd\psi)^2}{x(x-y)}+\frac{Y\,(d\phi+xd\psi)^2}{y(y-x)}\Big] \\ &\quad +\frac{a_0}{xy}\Big(d\phi+(x+y)d\psi+xydt\Big)^2 \end{aligned}$$

is Ricci flat, provided that X and Y are given by

$$\begin{aligned} X &= a_0+a_3\,x+a_2\,x^2+a_1\,x^3+a_0\,x^4 \\ Y &= a_0+a_1\,y+a_2\,y^2+a_3\,y^3+a_0\,y^4 \end{aligned}$$

Contain Myers-Perry BH and 1-rotation Black Ring as special cases. Generically, they describe genuine $D=5$ Taub-NUT type solutions (periodic time). A static limit describes black holes with lens-space boundary and horizon.

The Einstein Condition in Higher Dimensions

In d dimensions, the Riemann tensor has $\frac{1}{12}d^2(d^2-1)$ components, while the Ricci tensor has $\frac{1}{2}d(d+1)$. Thus the Einstein equation $R_{ij}=\lambda g_{ij}$ constrains the full Riemann curvature less and less as d increases. (In $d=3$ it determines the Riemann curvature completely.)

This is one reason why the solution space gets richer as the dimension increases. An instructive illustration is provided by looking at Einstein metrics on group manifolds.

Consider a simple compact Lie group G whose algebra is generated by T_a, with $[T_a,T_b]=f^c{}_{ab}\,T_c$. Then if g is in G, we defined left-invariant 1-forms σ^a by

$$g^{-1}\,dg=\sigma^a\,T_a\,,\qquad \text{satisfying}\qquad d\sigma^a=-\tfrac{1}{2}f^a{}_{bc}\,\sigma^b\wedge\sigma^c$$

These are invariant under the left action G_L of G: $g\longrightarrow Ag$.

For any group G, we can construct the bi-invariant metric (necessarily Einstein)

$$ds^2=\mathrm{tr}\,(g^{-1}\,dg)^2=\sigma_a\,\sigma_a$$

Except for the three-dimensional case $G=SU(2)$ (or $SO(3)$), it is known that every compact simple Lie group admits at least one additional inequivalent homogeneous Einstein metric, which is invariant under G_L but not under the full G_R (D'Atri and Ziller). Beyond this result, not much is known in general.

Einstein Metrics on Group Manifolds

The general left-invariant metric on G can be written as

$$ds^2 = x_{ab}\, \sigma_a\, \sigma_b$$

where x_{ab} is a symmetric constant tensor. The Einstein equations $R_{ab} = \lambda\, g_{ab}$ give a system of coupled polynomial equations for the x_{ab}.

Although in principle it is just a mechanical exercise to obtain these equations, it is just too complicated in general to try to solve them, even for the first example beyond $SU(2)$, namely $SU(3)$ (eight dimensional).

In recent work (Gibbons, Lü, CNP), we looked at three low(ish) dimensional examples, $SU(3)$, $SO(5)$ and G_2. By making simplifying ansätze for the choice of x_{ab}, inspired by symmetries of subgroups, we were able to obtain new Einstein metrics on $SO(5)$ and G_2, over and above the two known metrics. In total, we obtained 4 inequivalent Einstein metrics on $SO(5)$, and 6 on G_2.

How can we be sure two Einstein metrics are inequivalent? A very useful technique is to construct dimensionless scalar invariants from the curvature. Two convenient ones are

$$I_1 = \lambda\, V^{2/d}\,, \qquad I_2 = |\text{Riem}|^2\, \lambda^{-2}$$

where V is the volume of the d-dimensional group manifold. We can take $V = \sqrt{\det(x_{ab})}$. If one of these invariants is different for two Einstein metrics, then the metrics are guaranteed to be inequivalent.

I have recently been looking a bit more systematically at Einstein metrics on the $SO(n)$ group manifolds. The record so far is 16 inequivalent Einstein metrics on $SO(10)$. By looking at examples a pattern emerges, indicating how the number might in general grow with the dimension.

Einstein Metrics on $SO(n)$

We can take the left-invariant 1-forms to be L_{AB} $(= -L_{BA})$, satisfying $dL_{AB} = L_{AC} \wedge L_{CB}$. The following gives a (presumably incomplete) set of restricted metric ansätze that yield Einstein metrics:

1. The bi-invariant metric $ds^2 = \sigma_a\, \sigma_a$
2. Metrics adapted to the $SO(p) \times SO(q)$ subgroup, $p+q=n$, with $p \geq 3$ and $q \geq 3$. Each distinct such subgroup yields 3 inequivalent Einstein metrics within the class

$$ds^2 = x_1 \sum_{SO(p)} L_{ij}^2 + x_2 \sum_{SO(q)} L_{ab}^2 + x_3 \sum_{pq} L_{ia}^2$$

3. Higher factorisations into subgroups such as $SO(p) \times SO(q) \times SO(r)$ with $p+q+r=n$ and $p, q, r \geq 3$.

$$ds^2 = x_1 \sum_{SO(p)} L_{ij}^2 + x_2 \sum_{SO(q)} L_{ab}^2 + x_3 \sum_{SO(r)} L_{\alpha\beta}^2 + x_4 \sum_{p,q} L_{ia}^2 + x_5 \sum_{p,r} L_{i\alpha}^2 + x_6 \sum_{q,r} L_{a\alpha}^2$$

4. Grassmannian-type subgroups, such as where $A = (1, 2, i)$ and

$$ds^2 = x_1 \sum L_{1i}^2 + x_2 \sum L_{2i}^2 + x_3 \sum L_{ij}^2 + x_4 L_{12}^2$$

   There are 3 further inequivalent Einstein metrics in this class.

   The count so far is

$SO(5)$	$SO(6)$	$SO(7)$	$SO(8)$	$SO(9)$	$SO(10)$
4	5	7	8	10	16

Einstein Metrics on Spheres

The "floppiness" of the Einstein equations in higher dimensions is actually quite large. As shown by C. Bohm, any sphere S^d with $d \geq 5$ admits an infinite number of Einstein metrics. We can take $d = 5$ as an example.

The standard metric on S^5 can be written as

$$ds^2 = dr^2 + \sin^2 r\, d\Omega^2 + \cos^2 r\, d\widetilde{\Omega}^2\,, \qquad 0 \leq r \leq \frac{\pi}{2}$$

where $d\Omega^2$ and $d\widetilde{\Omega}^2$ are the metrics on a pair of unit 2-spheres.

Bohm's metrics, which have cohomogeneity 1, are constructed by making the ansatz

$$ds^2 = dr^2 + a(r)^2\, d\Omega^2 + b(r)^2\, d\widetilde{\Omega}^2$$

and solving the Einstein equations for the functions $a(r)$ and $b(r)$. These are

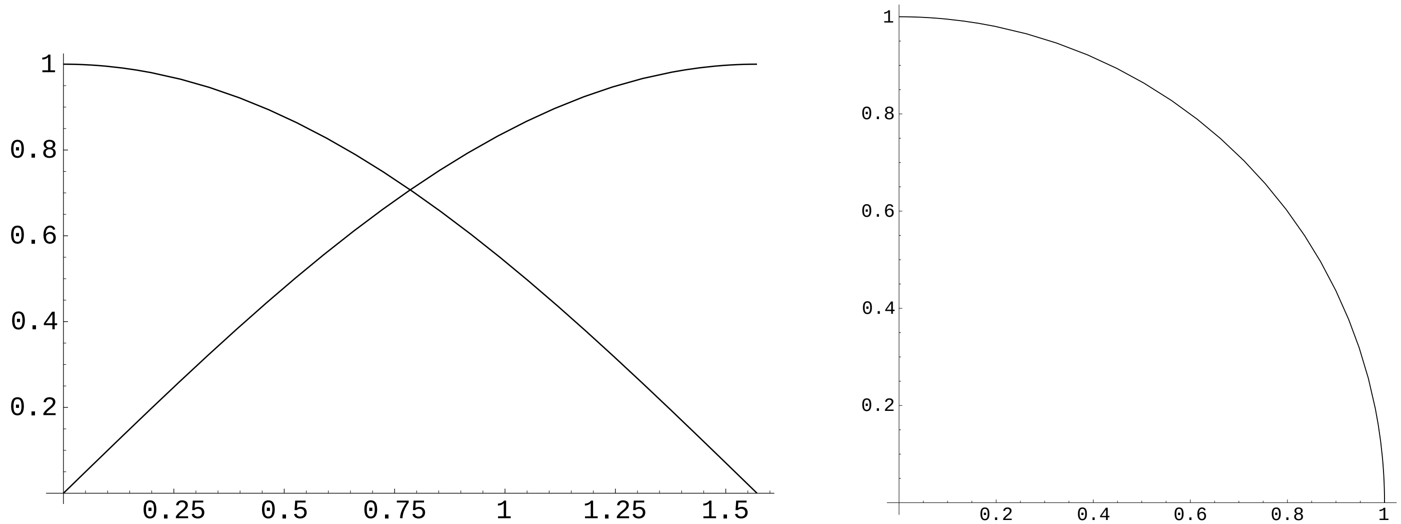
$$\frac{a''}{a} + \frac{2a'b'}{ab} + \frac{(a'^2 - 1)}{a^2} + \lambda = 0\,, \qquad \frac{b''}{b} + \frac{2a'b'}{ab} + \frac{(b'^2 - 1)}{b^2} + \lambda = 0\,,$$
$$\frac{(a'^2 - 1)}{a^2} + \frac{(b'^2 - 1)}{b^2} + \frac{4a'b'}{ab} + \frac{3\lambda}{2} = 0\,.$$

The desired solutions have $r_1 \leq r \leq r_2$, and for regularity at the endpoints they must satisfy essentially the same boundary conditions as $\sin r$ and $\cos r$:

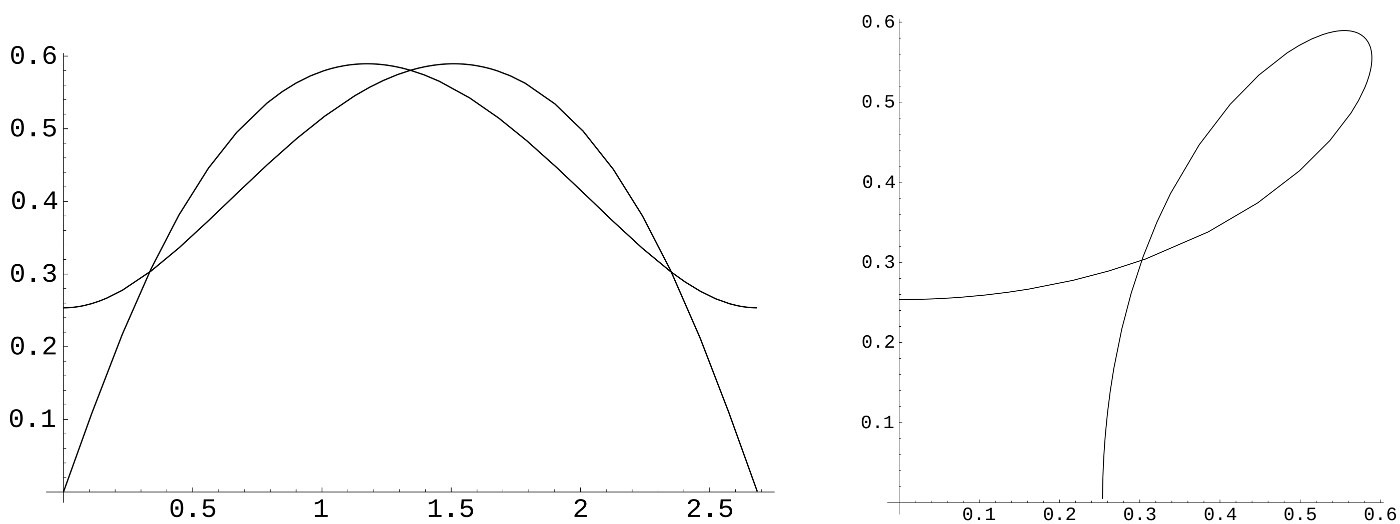
$$\begin{aligned} a(r_1) &= 0\,, & a'(r_1) &= 1\,, & b(r_1) &= b_0\,, & b'(r_1) &= 0\,, \\ a(r_2) &= a_0\,, & a'(r_2) &= 0\,, & b(r_2) &= 0\,, & b'(r_2) &= -1\,. \end{aligned}$$

The equations for a and b cannot be solved explicitly, and Bohm gave an "epsilon and delta" analysis proving the existence of a countable infinity of solutions satisfying the boundary conditions.

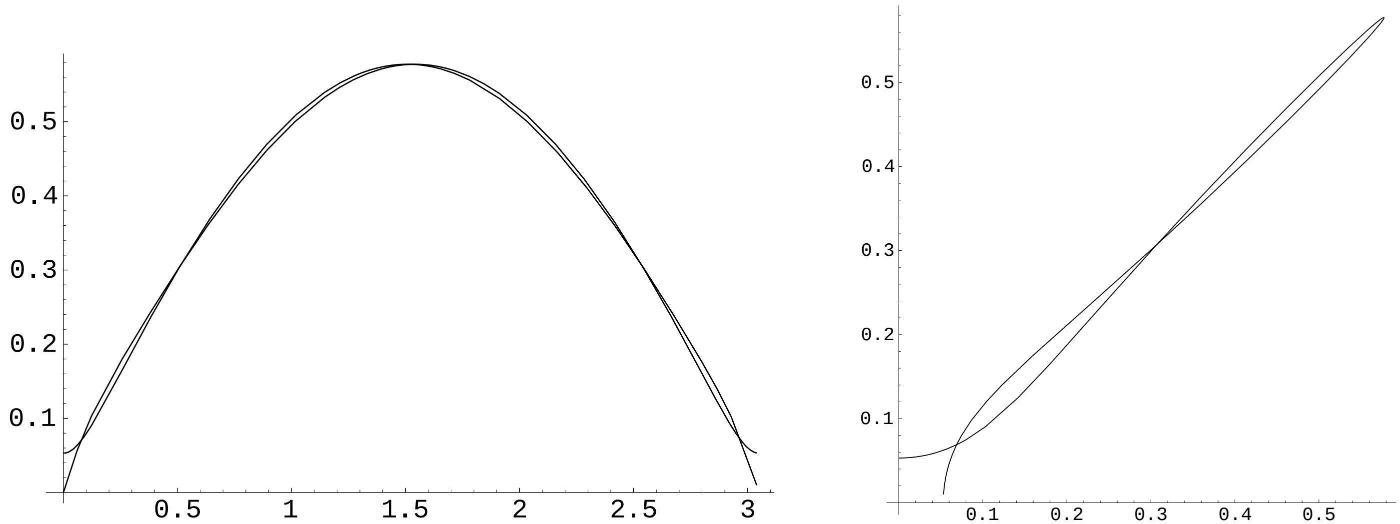
In (Gibbons, Hartnoll, CNP), we studied the solutions numerically. Here are some plots



The standard metric on S^5.



The first S^5 Einstein metric in the Bohm sequence.



The second S^5 Einstein metric in the Bohm sequence.

The sequence continues, *ad infinitum*...

These Einstein metrics on S^d with $d \geq 5$ can replace the usual round-sphere metric in higher-dimensional black holes. It seems they all have negative Lichnerowicz modes, and thus the "Bohm black holes" will be unstable.

Just as the usual S^5 can be analytically continued to give five-dimensional de Sitter spacetime dS_5, one can also analytically continue the Bohm metrics to give Lorentzian-signature Einstein metrics that generalise dS_5. They appear to be unstable to decay into the standard dS_5. The Bohm metrics have a totally-geodesic hypersurface, allowing them to be viewed as real tunnelling geometries for creating Lorentzian Bohm metrics "from nothing."

Further Remarks

- Looking at general relativity in higher dimensions opens up many new avenues for investigation, including non-standard horizon topologies (black rings, ...).
- No analytical results exist for charged rotating black holes in pure Einstein-Maxwell theory; black rings in an asymptotically AdS background; black rings in higher than five dimensions.
- String theory and M-theory provide a framework within which the investigation of higher-dimensional general relativity becomes important. For example, Ricci-flat metrics in dimensions $D \leq 11$, and Einstein metrics in $D = 5$ and $D = 7$ (AdS/CFT correspondence).
- The Einstein equations provide less of a constraint on the geometry in higher dimensions. In consequence, there can be more Einstein metrics on a given topology in higher dimensions. Some explicit examples include multiple homogeneous Einstein metrics on group manifolds, and countable infinities of inhomogeneous Einstein metrics on spheres and de Sitter-like spactimes.
- Understanding GR in higher dimensions can also provide interesting insights into what is generic versus what is particular to four dimensions.