THE BLACK HOLE/QUBIT CORRESPONDENCE

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Abstract

- Quantum entanglement lies at the heart of quantum information theory, with applications to quantum computing, teleportation, cryptography and communication. In the apparently separate world of quantum gravity, the Bekenstein-Hawking entropy of black holes has also occupied center stage.
- Here we describe a correspondence between the entanglement measures of qubits in quantum information theory and black hole entropy in string theory.

ADM

ADM

Schwarzschild

- 1972 PhD thesis problem: generate tree graphs for the Schwarzschild solution (Salam's bet with Bondi) and then include loop corrections. Puzzled to discover that with point source

$$g^{1/2}T_0{}^0 = \frac{16\pi M_0}{r^2}\delta(r)$$

even tree graphs divergent!

- Solution: Spherical shell of pressure-free dust

$$g^{1/2}T_0{}^0 = \frac{16\pi M_0}{r^2}\delta(r-\epsilon)$$

ADM Physical Review Letters 4, 375, 1960

Renormalize

- Absorb infinity at $\epsilon = 0$ into a mass renormalization

$$M = M_0 - \frac{1}{2}\frac{{M_0}^2}{\epsilon}$$

- Note that in isotropic coordinates

$$\epsilon = \left(1 + \frac{M}{2\epsilon_I}\right)\epsilon_I$$

and

$$M = M_0 - \frac{1}{2}\frac{M^2}{\epsilon_I}$$

Bondi: "equivalence principle"

Quantum corrections to Schwarzschild

- One-loop corrections Duff, Phys. Rev. D9, 1837, 1974:

$$g_{00} = -1 + \frac{2GM}{r} + \alpha\frac{\hbar G^2 M}{r^3}$$

For CFT loops: $45\pi\alpha = 12N_1 + 3N_{1/2} + N_0$

- Fast forward 25 years: R-S braneworld

$$g_{00} = -1 + \frac{2GM}{r} + \frac{2L^2}{3r^3} \qquad L = \frac{2G_5}{G}$$

RS Phys. Rev. Lett. 83, 4690,1999

- AdS/CFT miracle: For $U(N)$ super-Yang-Mills $3\pi\alpha = 2N^2$ and $\hbar N^2 = \pi L^3/2G_5$, so $D = 4$ quantum result same as $D = 5$ classical result.
  Duff Liu, Phys. Rev. Lett. 85, 2052, 2000

Qubits

QUBITS

Two qubits

- The two qubit system Alice and Bob (where $A, B = 0, 1$) is described by the state

$$\begin{aligned} |\Psi\rangle &= a_{AB}|AB\rangle \\ &= a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle. \end{aligned}$$

- The bipartite entanglement of Alice and Bob is given by

$$\tau_{AB} = 4|\det \rho_A| = 4|\det a_{AB}|^2,$$

  where

$$\rho_A = Tr_B|\Psi\rangle\langle\Psi|$$

- τ_{AB} is invariant under $SL(2)_A \times SL(2)_B$, with a_{AB} transforming as a $(2, 2)$, and under a discrete duality that interchanges A and B.

Two qubits: examples

- Example, separable state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

$$\tau_{AB} = 0$$

- Example, Bell state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$\tau_{AB} = 1$$

- EPR "paradox"

Three qubits

The three qubit system Alice, Bob and Charlie (where $A, B, C = 0, 1$) is described by the state

$$
\begin{aligned}
|\Psi\rangle &= a_{ABC}|ABC\rangle \\
&= a_{000}|000\rangle + a_{001}|001\rangle + a_{010}|010\rangle + a_{011}|011\rangle \\
&+ a_{100}|100\rangle + a_{101}|101\rangle + a_{110}|110\rangle + a_{111}|111\rangle.
\end{aligned}
$$

Cayley's hyperdeterminant

- The tripartite entanglement of Alice, Bob and Charlie is given by

$$\tau_{ABC} = 4|\mathrm{Det}\ a_{ABC}|,$$

Coffman et al: quant-ph/9907047

- $\mathrm{Det}\ a_{ABC}$ is Cayley's hyperdeterminant

$$\mathrm{Det}\ a_{ABC} = -\frac{1}{2}\varepsilon^{A_1A_2}\varepsilon^{B_1B_2}\varepsilon^{C_1C_4}\varepsilon^{C_2C_3}\varepsilon^{A_3A_4}\varepsilon^{B_3B_4} \cdot a_{A_1B_1C_1}a_{A_2B_2C_2}a_{A_3B_3C_3}a_{A_4B_4C_4}$$

Miyake and Wadati: quant-ph/0212146

Symmetry

- Explicitly

$$
\begin{aligned}
&\text{Det } a_{ABC} = \\
&a_{000}^2 a_{111}^2 + a_{001}^2 a_{110}^2 + a_{010}^2 a_{101}^2 + a_{100}^2 a_{011}^2 \\
&- 2(a_{000} a_{001} a_{110} a_{111} + a_{000} a_{010} a_{101} a_{111} \\
&+ a_{000} a_{100} a_{011} a_{111} + a_{001} a_{010} a_{101} a_{110} \\
&+ a_{001} a_{100} a_{011} a_{110} + a_{010} a_{100} a_{011} a_{101}) \\
&+ 4(a_{000} a_{011} a_{101} a_{110} + a_{001} a_{010} a_{100} a_{111}).
\end{aligned}
$$

- It is invariant under $SL(2)_A \times SL(2)_B \times SL(2)_C$, with a_{ABC} transforming as a $(2, 2, 2)$, and under a discrete triality that interchanges A, B and C.

Local entropy

Another useful quantity is the local entropy S_A, which is a measure of how entangled A is with the pair BC:

$$S_A = 4\mathrm{det}\ \rho_A \equiv \tau_{A(BC)}$$

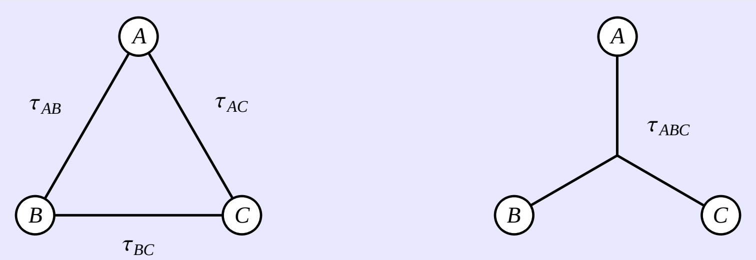
where ρ_A is the reduced density matrix

$$\rho_A = \mathrm{Tr}_{BC}|\Psi\rangle\langle\Psi|,$$

and with similar formulae for B and C.

Tangles

Tangles



- 2-tangles τ_{AB}, τ_{BC}, and τ_{CA} give bipartite entanglements between pairs in 3-qubit system
- 3-tangle τ_{ABC} is a measure of the genuine 3-way entanglement:

$$\tau_{ABC} = \tau_{A(BC)} - \tau_{AB} - \tau_{CA}$$

Entanglement classes

Entanglement classes

Class	$\tau_{A(BC)}$	$\tau_{B(AC)}$	$\tau_{(AB)C}$	τ_{ABC}
A-B-C	0	0	0	0
A-BC	0	> 0	> 0	0
B-CA	> 0	0	> 0	0
C-AB	> 0	> 0	0	0
W	> 0	> 0	> 0	0
GHZ	> 0	> 0	> 0	$\neq 0$

Dur, Vidal, Cirac: quant-ph/0005115

LOCC

- Local Operations and Classical Communication=LOCC
- Two states are said to be LOCC equivalent if and only if they may be transformed into one another with certainty using LOCC protocols. Reviews of the LOCC paradigm and entanglement measures may be found in Plenio:2007,Horodecki:2007.

Orbits

- Two states of a composite quantum system are regarded as LOCC equivalent if they are related by a unitary transformation which factorizes into separate transformations on the component parts (Bennett:1999), so-called *local unitaries*. The Hilbert space decomposes into equivalence classes, or *orbits* under the action of the group of local unitaries.
- In the case of n qubits the group of local unitaries is given (up to a global phase) by $[SU(2)]^n$.

SLOCC

- Stochastic Local Operations and Classical Communication=SLOCC
- Two quantum states are said to be SLOCC equivalent if and only if they may be transformed into one another with some *non-vanishing probability* using LOCC operations (Bennett:1999, Dur:2000). The set of SLOCC transformations relating equivalent states forms a group (which we will refer to as the *SLOCC equivalence group*).
- For n qubits the SLOCC equivalence group is given (up to a global complex factor) by the n-fold tensor product, $[SL(2, C)]^n$, one factor for each qubit (Dur:2000). Note, the LOCC equivalence group forms a compact subgroup of the larger SLOCC equivalence group.

Complex qubit parameters

- For unnormalized three-qubit states, the number of parameters [Linden and Popescu: quant-ph/9711016] needed to describe inequivalent states or, what amounts to the same thing, the number of algebraically independent invariants [Sudbery: quant-ph/0001116] is given by the dimension of the space of orbits

$$\frac{\mathbb{C}^2 \times \mathbb{C}^2 \times \mathbb{C}^2}{U(1) \times SU(2) \times SU(2) \times SU(2)}$$

namely, $16 - 10 = 6$.

Real qubit parameters

- However, for subsequent comparison with the *STU* black hole [Duff, Liu and Rahmfeld: hep-th/9508094; Behrndt et al: hep-th/9608059], we restrict our attention to states with *real* coefficients a_{ABC}.
- In this case, one can show that there are five algebraically independent invariants: $\mathrm{Det}\ \mathrm{a}$, S_A, S_B, S_C and the norm $\langle\Psi|\Psi\rangle$, corresponding to the dimension of

$$\frac{\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2}{SO(2) \times SO(2) \times SO(2)}$$

namely, $8 - 3 = 5$.

5 parameter state

- Hence, the most general real three-qubit state can be described by just five parameters.
  Acin et al: quant-ph/0009107
- It may conveniently be written

$$|\Psi\rangle = -N_3 \cos^2\theta|001\rangle - N_2|010\rangle + N_3 \sin\theta\cos\theta|011\rangle - N_1|100\rangle - N_3 \sin\theta\cos\theta|101\rangle + (N_0 + N_3 \sin^2\theta)|111\rangle.$$

Representatives

Representatives from each class are:

- Class A-B-C (product states):

$$N_0|111\rangle.$$

- Classes A-BC, (bipartite entanglement):

$$N_0|111\rangle - N_1|100\rangle,$$

  and similarly B-CA, C-AB.

- Class W (maximizes bipartite entanglement):

$$-N_1|100\rangle - N_2|010\rangle - N_3|001\rangle.$$

- Class GHZ (genuine tripartite entanglement):

$$N_0|111\rangle - N_1|100\rangle - N_2|010\rangle - N_3|001\rangle.$$

STU black holes

STU BLACK HOLES

STU model

The *STU* model consists of $N = 2$ supergravity coupled to three vector multiplets interacting through the special Kahler manifold $[SL(2)/SO(2)]^3$:

$$\mathcal{S}_{\mathrm{STU}} = \frac{1}{16\pi G}\int \mathrm{e}^{-\eta}\Big[$$

$$\left(R + \frac{1}{4}\left(\mathrm{Tr}\left[\partial\mathcal{M}_T^{-1}\partial\mathcal{M}_T\right] + \mathrm{Tr}\left[\partial\mathcal{M}_U^{-1}\partial\mathcal{M}_U\right]\right)\right)\star 1$$

$$+\star\,\mathrm{d}\eta\wedge\mathrm{d}\eta - \frac{1}{2}\star H_{[3]}\wedge H_{[3]} - \frac{1}{2}\star F_{S[2]}^{\mathrm{T}}\wedge(\mathcal{M}_T\otimes\mathcal{M}_U)\,F_{S[2]}\Big]$$

$$\mathcal{M}_S = \frac{1}{\Im(S)}\begin{pmatrix} 1 & \Re(S) \\ \Re(S) & |S|^2 \end{pmatrix} \quad \text{etc.}$$

STU parameters

- A general static spherically symmetric black hole solution depends on 8 charges denoted $q_0, q_1, q_2, q_3, p^0, p^1, p^2, p^3$, but the generating solution depends on just $8 - 3 = 5$ parameters [Cvetic and Youm: hep-th/9512127; Cvetic and Hull: hep-th/9606193], after fixing the action of the isotropy subgroup $[SO(2)]^3$.

Black hole entropy

- Black hole entropy S given by the one quarter the area of the event horizon.
  Hawking: 1975
- The STU black hole entropy is a complicated function of the 8 charges :

$$(S/\pi)^2 = -(p \cdot q)^2$$
$$+4\Big[(p^1q_1)(p^2q_2) + (p^1q_1)(p^3q_3) + (p^3q_3)(p^2q_2)$$
$$+q_0p^1p^2p^2 - p^0q_1q_2q_3\Big]$$

Behrndt et al: hep-th/9608059

Qubit correspondence

- By identifying the 8 charges with the 8 components of the three-qubit hypermatrix a_{ABC},

$$\begin{bmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \\ q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} a_{000} \\ -a_{001} \\ -a_{010} \\ -a_{100} \\ a_{111} \\ a_{110} \\ a_{101} \\ a_{011} \end{bmatrix}$$

one finds that the black hole entropy is related to the 3-tangle as in

$$S = \pi\sqrt{|\mathrm{Det}\ a_{ABC}|} = \frac{\pi}{2}\sqrt{\tau_{ABC}}$$

Duff: hep-th/0601134

BH/qubit correspondence

- The measure of tripartite entanglement of three qubits (Alice, Bob and Charlie), known as the 3-tangle τ_{ABC}, and the entropy S of the 8-charge STU black hole of supergravity are both given by Cayley's hyperdeterminant.

Further developments

- Further papers have written a more complete dictionary, which translates a variety of phenomena in one language to those in the other:

Further developments contd

- The attractor mechanism on the black hole side is related to optimal local distillation protocols on the QI side.
- Moreover, supersymmetric and non-supersymmetric black holes corresponding to the suppression or non-suppression of bit-flip errors .

Levay:[arXiv:0708.2799 [hep-th]]

$N = 8$ GENERALIZATION

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Supergravity in $D \leq 11$

D	scalars/vectors	G	H
10A	1 / 1	$SO(1,1,R)$	–
10B	2 / 0	$SL(2,R)$	$SO(2,R)$
9	3 / 3	$SL(2,R) \times SO(1,1,R)$	$SO(2,R)$
8	7 / 6	$SL(2,R) \times SL(3,R)$	$SO(2,R) \times SO(3,R)$
7	14 / 10	$SL(5,R)$	$SO(5,R)$
6	25 / 16	$SO(5,5,R)$	$SO(5,R) \times SO(5,R)$
5	42 / 27	$E_{6(6)}(R)$	$USP(8)$
4	70 / 28	$E_{7(7)}(R)$	$SU(8)$
3	128/ 0	$E_{8(8)}(R)$	$SO(16,R)$

Table: The symmetry groups (G) of the low energy supergravity theories with 32 supercharges in different dimensions (D) and their maximal compact subgroups (H).

Embeddings

The $N = 2$ *STU* solution can usefully be embedded in

- $N = 4$ supergravity with symmetry $SL(2) \times SO(6, 22)$, where the charges transform as a $(2, 28)$.
- $N = 8$ supergravity with symmetry $E_{7(7)}$, where the charges transform as a 56.

Remarkably, the same five parameters suffice to describe these 56-charge black holes.

$E_{7(7)}$ and seven qubits

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$E_{7(7)}$

- There is, in fact, a quantum information theoretic interpretation of the 56 charge $N = 8$ black hole in terms of a Hilbert space consisting of seven copies of the three-qubit Hilbert space. It relies on the decomposition $E_{7(7)} \supset [SL(2)]^7$

Decomposition of the 56

- Under

$$E_{7(7)} \supset$$

$$SL(2)_A \times SL(2)_B \times SL(2)_C \times SL(2)_D \times SL(2)_E \times SL(2)_F \times SL(2)_G$$

the 56 decomposes as

$$
\begin{aligned}
56 \rightarrow \\
&(2,2,1,2,1,1,1) \\
&+(1,2,2,1,2,1,1) \\
&+(1,1,2,2,1,2,1) \\
&+(1,1,1,2,2,1,2) \\
&+(2,1,1,1,2,2,1) \\
&+(1,2,1,1,1,2,2) \\
&+(2,1,2,1,1,1,2)
\end{aligned}
$$

Seven qubits

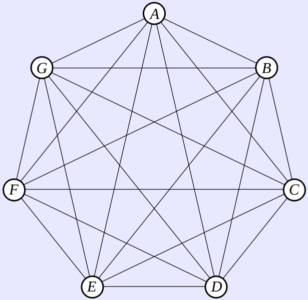
- It admits the interpretation of a tripartite entanglement of seven qubits, Alice, Bob, Charlie, Daisy, Emma, Fred and George:

$$
\begin{aligned}
|\psi\rangle = \quad & a_{ABD}|ABD\rangle \\
& + b_{BCE}|BCE\rangle \\
& + c_{CDF}|CDF\rangle \\
& + d_{DEG}|DEG\rangle \\
& + e_{EFA}|EFA\rangle \\
& + f_{FGB}|FGB\rangle \\
& + g_{GAC}|GAC\rangle
\end{aligned}
$$

E_7 Entanglement

The following diagram may help illustrate the tripartite entanglement between the 7 qubits

E_7 Entanglement



Cartan invariant

- The entanglement measure given by Cartan's quartic $E_{7(7)}$ invariant.

$$I_4 = -\mathrm{Tr}((xy)^2) + \frac{1}{4}\mathrm{Tr}(xy)^2 - 4\,(\mathrm{Pf}(x) + \mathrm{Pf}(y))$$

x^{IJ} and y_{IJ} are again 8×8 antisymmetric charge matrices
Duff and Ferrara: quant-ph/0609227
Levay: hep-th/0610314

x^{IJ}

$$x^{IJ} =$$

$$\begin{pmatrix}
0 & -a_{111} & -b_{111} & -c_{111} & -d_{111} & -e_{111} & -f_{111} & -g_{111} \\
a_{111} & 0 & f_{001} & d_{100} & -c_{010} & g_{010} & -b_{100} & -e_{001} \\
b_{111} & -f_{001} & 0 & g_{001} & e_{100} & -d_{010} & a_{010} & -c_{100} \\
c_{111} & -d_{100} & -g_{001} & 0 & a_{001} & f_{100} & -e_{010} & b_{010} \\
d_{111} & c_{010} & -e_{100} & -a_{001} & 0 & b_{001} & g_{100} & -f_{010} \\
e_{111} & -g_{010} & d_{010} & -f_{100} & -b_{001} & 0 & c_{001} & a_{100} \\
f_{111} & b_{100} & -a_{010} & e_{010} & -g_{100} & -c_{001} & 0 & d_{001} \\
g_{111} & e_{001} & c_{100} & -b_{010} & f_{010} & -a_{100} & -d_{001} & 0
\end{pmatrix}$$

y_{IJ}

$$y_{IJ} =$$

$$\begin{pmatrix} 0 & -a_{000} & -b_{000} & -c_{000} & -d_{000} & -e_{000} & -f_{000} & -g_{000} \\ a_{000} & 0 & f_{110} & d_{011} & -c_{101} & g_{101} & -b_{011} & -e_{110} \\ b_{000} & -f_{110} & 0 & g_{110} & e_{011} & -d_{101} & a_{101} & -c_{011} \\ c_{000} & -d_{011} & -g_{110} & 0 & a_{110} & f_{011} & -e_{101} & b_{101} \\ d_{000} & c_{101} & -e_{011} & -a_{110} & 0 & b_{110} & g_{011} & -f_{101} \\ e_{000} & -g_{101} & d_{101} & -f_{011} & -b_{110} & 0 & c_{110} & a_{011} \\ f_{000} & b_{011} & -a_{101} & e_{101} & -g_{011} & -c_{110} & 0 & d_{110} \\ g_{000} & e_{110} & c_{011} & -b_{101} & f_{101} & -a_{011} & -d_{110} & 0 \end{pmatrix}$$

I_4

Schematically,

$$
\begin{aligned}
I_4 = a^4 + b^4 + c^4 + d^4 + e^4 + f^4 + g^4 \\
+ 2\Big[a^2b^2 &+ a^2c^2 + a^2d^2 + a^2e^2 + a^2f^2 + a^2g^2 \\
&+ b^2c^2 + b^2d^2 + b^2e^2 + b^2f^2 + b^2g^2 \\
&+ c^2d^2 + c^2e^2 + c^2f^2 + c^2g^2 \\
&+ d^2e^2 + d^2f^2 + d^2g^2 \\
&+ e^2f^2 + e^2g^2 \\
&+ f^2g^2\Big] \\
+ 8\,[abce + bcdf + cdeg + defa + efgb + fgac + gabd]\,,
\end{aligned}
$$

where a^4 is Cayley's hyperdeterminant etc

$N = 8$ case

- Remarkably, because the generating solution depends on the same five parameters as the *STU* model, its classification of states will exactly parallel that of the usual three qubits. Indeed, the Cartan invariant reduces to Cayley's hyperdeterminant in a canonical basis.

Kallosh and Linde: hep-th/0602061

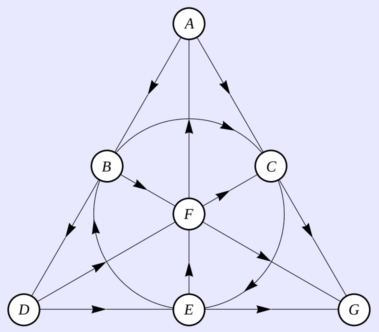
OCTONIONS AND THE FANO PLANE

OCTONIONS AND THE FANO PLANE

Fano plane

An alternative description is provided by the Fano plane which has seven points, representing the seven qubits, and seven lines (the circle counts as a line) with three points on every line, representing the tripartite entanglement, and three lines through every point.

Fano plane



Octonions

The Fano plane also provides the multiplication for the imaginary octonions:

	A	B	C	D	E	F	G
A		D	G	$-B$	F	$-E$	$-C$
B	$-D$		E	A	$-C$	G	$-F$
C	$-G$	$-E$		F	B	$-D$	A
D	B	$-A$	$-F$		G	C	$-E$
E	$-F$	C	$-B$	$-G$		A	D
F	E	$-G$	D	$-C$	$-A$		B
G	C	F	$-A$	E	$-D$	$-B$	

CLASSIFICATION

CLASSIFICATION

CLASSIFICATION

- Furthermore, one can relate the classification of three-qubit entanglements to the classification of supersymmetric black holes as in the following table:

Table

Class	S_A	S_B	S_C	Det a	Black hole	Susy
A-B-C	0	0	0	0	small	1/2
A-BC	0	> 0	> 0	0	small	1/4
B-CA	> 0	0	> 0	0	small	1/4
C-AB	> 0	> 0	0	0	small	1/4
W	> 0	> 0	> 0	0	small	1/8
GHZ	> 0	> 0	> 0	< 0	large	1/8
GHZ	> 0	> 0	> 0	> 0	large	0

Table: Classification of three-qubit entanglements and their corresponding $D = 4$ black holes.

Wrapped D3-branes and 3 qubits

WRAPPED D3-BRANES AND 3 QUBITS

Microscopic analysis

String interpretation:

- $N = 4$ supergravity with symmetry $SL(2) \times SO(6, 22)$ is the low-energy limit of the heterotic string compactified on T^6.
- $N = 8$ supergravity with symmetry $E_{7(7)}$ is the low-energy limit of the Type IIA or Type IIB strings, compactified on T^6 or M-theory on T^7.
- Black holes are now 0-branes obtained by wrapping p-branes around p of the compactifying circles.

Stringy version

- The stringy version of the STU black hole is not unique since there are many ways of embedding the *STU* model in string/M-theory, but a useful one from our point of view is that of four D3-branes wrapping the $(579), (568), (478), (469)$ cycles of T^6 (intersecting over a string) with wrapping numbers N_0, N_1, N_2, N_3.
Klebanov and Tseytlin: hep-th/9604166
- The wrapped circles are denoted by a cross and the unwrapped circles by a nought as shown in the following table:

4	5	6	7	8	9	macro charges	micro charges	$\lvert ABC\rangle$
x	o	x	o	x	o	p^0	0	$\lvert 000\rangle$
o	x	o	x	x	o	q_1	0	$\lvert 110\rangle$
o	x	x	o	o	x	q_2	$-N_3 \sin\theta\cos\theta$	$\lvert 101\rangle$
x	o	o	x	o	x	q_3	$N_3 \sin\theta\cos\theta$	$\lvert 011\rangle$
o	x	o	x	o	x	q_0	$N_0 + N_3 \sin^2\theta$	$\lvert 111\rangle$
x	o	x	o	o	x	$-p^1$	$-N_3 \cos^2\theta$	$\lvert 001\rangle$
x	o	o	x	x	o	$-p^2$	$-N_2$	$\lvert 010\rangle$
o	x	x	o	x	o	$-p^3$	$-N_1$	$\lvert 100\rangle$

Table: Three qubit interpretation of the 8-charge $D = 4$ black hole from four D3-branes wrapping around the lower four cycles of T^6 with wrapping numbers N_0, N_1, N_2, N_3.

Fifth parameter

- The fifth parameter θ is obtained by allowing the N_3 brane to intersect at an angle which induces additional effective charges on the $(579), (569), (479)$ cycles [Balasubramanian and Larsen: hep-th/ 9704143; Balasubramanian: hep-th/9712215; Bertolini and Trigiante: hep-th/0002191].
- The microscopic calculation of the entropy consists of taking the logarithm of the number of microstates and yields the same result as the macroscopic one [Bertolini and Trigiante: hep-th/0008201].

Qubit interpretation

- To make the black hole/qubit correspondence we associate the three T^2 with the $SL(2)_A \times SL(2)_B \times SL(2)_C$ of the three qubits Alice, Bob, and Charlie. The 8 different cycles then yield 8 different basis vectors $|ABC\rangle$ as in the last column of the Table, where $|0\rangle$ corresponds to xo and $|1\rangle$ to ox.
- We see immediately that we reproduce the five parameter three-qubit state $|\Psi\rangle$:

$$|\Psi\rangle = -N_3\cos^2\theta|001\rangle - N_2|010\rangle + N_3\sin\theta\cos\theta|011\rangle - N_1|100\rangle - N_3\sin\theta\cos\theta|101\rangle + (N_0 + N_3\sin^2\theta)|111\rangle.$$

- Note from the Table that the GHZ state describes four D3-branes intersecting over a string, or groups of 4 wrapping cycles with just one cross in common.

IIA and IIB

- Performing a T-duality transformation, one obtains a Type IIA interpretation with zero D6-branes, N_0 D0-branes, N_1, N_2, N_3 D4-branes plus effective D2-brane charges, where $|0\rangle$ now corresponds to xx and $|1\rangle$ to oo.

SUMMARY

Summary

- Our Type IIB microscopic analysis of the $D = 4$ black hole has provided an explanation for the appearance of the qubit two-valuedness (0 or 1) that was lacking in the previous treatments: The brane can wrap one circle or the other in each T^2.
  To wrap or not to wrap? That is the qubit.
- The number of qubits is three because of the number of extra dimensions is six.
- The five parameters of the real three-qubit state are seen to correspond to four D3-branes intersecting at an angle.

Wrapped D3-branes and 3 qubits

WRAPPED M2-BRANES AND 2 QUTRITS

Qutrit interpretation

- All this suggests that the analogy between $D = 5$ black holes and three-state systems (0 or 1 or 2), known as qutrits [Duff and Ferrara: 0704.0507 [hep-th]], should involve the choice of wrapping a brane around one of three circles in T^3. This is indeed the case, with the number of qutrits being two.
- The two-qutrit system (where $A, B = 0, 1, 2$) is described by the state

$$|\Psi\rangle = a_{AB}|AB\rangle,$$

and the Hilbert space has dimension $3^2 = 9$.

2-tangle

- The bipartite entanglement of Alice and Bob is given by the 2-tangle
$$\tau_{AB} = 27\det \rho_A = 27|\det a_{AB}|^2,$$
where ρ_A is the reduced density matrix
$$\rho_A = \mathrm{Tr}_B|\Psi\rangle\langle\Psi|.$$
- The determinant is invariant under $SL(3)_A \times SL(3)_B$, with a_{AB} transforming as a $(3, 3)$, and under a discrete duality that interchanges A and B.

$D=5$ black hole

- For subsequent comparison with the $D=5$ black hole, we restrict our attention to unnormalized states with real coefficients a_{AB}.
- There are three algebraically independent invariants : τ_{AB}, C_2 (the sum of the principal minors of ρ_{AB}) and the norm $\langle\Psi|\Psi\rangle$, corresponding to the dimension of

$$\frac{\mathbb{R}^3\times\mathbb{R}^3}{SO(3)\times SO(3)}$$

namely, $9-6=3$.

$D = 5$ black hole

- Hence, the most general two-qutrit state can be described by just three parameters, which may conveniently taken to be three real numbers N_0, N_1, N_2,.

$$|\Psi\rangle = N_0|00\rangle + N_1|11\rangle + N_2|22\rangle$$

- A classification of two-qutrit entanglements, depending on the rank of the density matrix, is given in the following table:

$D = 5$ table

Class	C_2	τ_{AB}	Black hole	Susy
A-B	0	0	small	1/2
Rank 2 Bell	> 0	0	small	1/4
Rank 3 Bell	> 0	> 0	large	1/8

Table: Classification of two-qutrit entanglements and their corresponding $D = 5$ black holes.

$D = 5$ black hole

- The 9-charge $N = 2$, $D = 5$ black hole may also be embedded in the $N = 8$ theory in different ways. The most convenient microscopic description is that of three M2-branes wrapping the (58), (69), (710) cycles of the T^6 compactification of $D = 11$ M-theory, with wrapping numbers N_0, N_1, N_2 and intersecting over a point [Papadopoulos and Townsend: hep-th/9603087; Klebanov and Tseytlin:hep-th/9604166].
- To make the black hole/qutrit correspondence we associate the two T^3 with the $SL(3)_A \times SL(3)_B$ of the two qutrits Alice and Bob, where $|0\rangle$ corresponds to xoo, $|1\rangle$ to oxo and $|2\rangle$ to oox.The 9 different cycles then yield the 9 different basis vectors $|AB\rangle$ as in the last column of the following Table:

$D = 5$ table

5	6	7	8	9	10	macro charges	micro charges	$\|AB\rangle$
x	o	o	x	o	o	p^0	N_0	$\|00\rangle$
o	x	o	o	x	o	p^1	N_1	$\|11\rangle$
o	o	x	o	o	x	p^2	N_2	$\|22\rangle$
x	o	o	o	x	o	p^3	0	$\|01\rangle$
o	x	o	o	o	x	p^4	0	$\|12\rangle$
o	o	x	x	o	o	p^5	0	$\|20\rangle$
x	o	o	o	o	x	p^6	0	$\|02\rangle$
o	x	o	x	o	o	p^7	0	$\|10\rangle$
o	o	x	o	x	o	p^8	0	$\|21\rangle$

- We see immediately that we reproduce the three parameter two-qutrit state $|\Psi\rangle$:

$$|\Psi\rangle = N_0|00\rangle + N_1|11\rangle + N_2|22\rangle$$

- The black hole entropy, both macroscopic and microscopic, turns out to be given by the 2-tangle

$$S = 2\pi\sqrt{|\det a_{AB}|},$$

  and the classification of the two-qutrit entanglements matches that of the black holes .

- Note that the non-vanishing cubic combinations appearing in $\det a_{AB}$ correspond to groups of 3 wrapping cycles with no crosses in common, i.e. that intersect over a point.

Embeddings

- There is, in fact, a quantum information theoretic interpretation of the 27 charge $N = 8, D = 5$ black hole in terms of a Hilbert space consisting of three copies of the two-qutrit Hilbert space. It relies on the decomposition $E_{6(6)} \supset [SL(3)]^3$ and admits the interpretation of a bipartite entanglement of three qutrits, with the entanglement measure given by Cartan's cubic $E_{6(6)}$ invariant.
  Duff and Ferrara: 0704.0507 [hep-th]
- Once again, however, because the generating solution depends on the same three parameters as the 9-charge model, its classification of states will exactly parallel that of the usual two qutrits. Indeed, the Cartan invariant reduces to $\det a_{AB}$ in a canonical basis.
  Ferrara and Maldacena: hep-th/9706097

Summary

- Our M-theory analysis of the $D = 5$ black hole has provided an explanation for the appearance of the qutrit three-valuedness (0 or 1 or 2) that was lacking in the previous treatments: The brane can wrap one of the three circles in each T^3.
- The number of qutrits is two because of the number of extra dimensions is six.
- The three parameters of the real two-qutrit state are seen to correspond to three intersecting M2-branes.

RECENT DEVELOPMENTS

Recent developments

- Whether or not there is an underlying physical connection, this two-way process teaches us new things about both black holes and QI.
- Recent examples are:

FTS

- Correspondence between a three-qubit state vector ψ and a Freudenthal triple system Ψ over the Jordan algebra $C \oplus C \oplus C$:

$$\psi = a_{ABC}|ABC> \quad \leftrightarrow$$

$$\Psi = \begin{pmatrix} a_{111} & (a_{001}, a_{010}, a_{100}) \\ (a_{110}, a_{101}, a_{011}) & a_{000} \end{pmatrix}, \qquad (7.1)$$

the structure of the FTS naturally captures the SLOCC classification.
L. Borsten, D. Dahanayake, M. J. Duff, H. Ebrahim and W. Rubens, arXiv:0812.3322 [quant-ph].

Class	Rank	FTS rank condition	
		vanishing	non-vanishing
Null	0	Ψ	–
A-B-C	1	$3T(\Psi, \Psi, \Phi) + \{\Psi, \Phi\}\Psi$	Ψ
A-BC	2a	$T(\Psi, \Psi, \Psi)$	γ^A
B-CA	2b	$T(\Psi, \Psi, \Psi)$	γ^B
C-AB	2c	$T(\Psi, \Psi, \Psi)$	γ^C
W	3	$q(\Psi)$	$T(\Psi, \Psi, \Psi)$
GHZ	4	–	$q(\Psi)$

Table: The entanglement classification of three qubits as according to the FTS rank system

Orbits

Table: Coset spaces of the orbits of the 3-qubit state space $C^2 \times C^2 \times C^2$ under the action of the SLOCC group $[SL(2,C)]^3$.

Class	FTS Rank	Orbits	dim
Separable	1	$\frac{[SL(2,C)]^3}{[SO(2,C)]^2 \ltimes C^3}$	4
Bi-separable	2	$\frac{[SL(2,C)]^3}{O(3,C) \times C}$	5
W	3	$\frac{[SL(2,C)]^3}{C^2}$	7
GHZ	4	$\frac{[SL(2,C)]^3}{[SO(2,C)]^2}$	7

New duality for black holes; arXiv:0903.5517

- It is well-known that the quantized charges x of 4D black holes may be assigned to elements of an integral Freudenthal triple system (FTS) whose automorphism group is the corresponding U-duality. The FTS is equipped with a quartic form $\Delta(x)$ whose square root yields the lowest order black hole entropy.
- We show that a subset of these black holes, for which $\Delta(x)$ is necessarily a perfect square, admit a *Freudenthal dual* with integer charges $\tilde{x}$, for which $\tilde{\tilde{x}} = -x$ and $\Delta(\tilde{x}) = \Delta(x)$ Some, but not all, of other discrete U-duality invariants are also Freudenthal invariant.
- Similar story in 5D where we introduce a *Jordan dual* $A^{\star}$, for which $A^{\star\star} = A$ with cubic norm $N(A^{\star}) = N(A)$, whose square is necessarily a perfect cube.

Octonions and supersmmetry

- In the $\mathcal{N} = 8$ case, the **56** of $E_{7(7)}$ decomposes as

$$\mathbf{56} \to (\mathbf{2}, \mathbf{12}) + (\mathbf{1}, \mathbf{32}), \tag{7.2}$$

under

$$E_{7(7)} \supset SL(2) \times SO(6,6) \tag{7.3}$$

where $SL(2)$ is the electric-magnetic S-duality and $SO(6,6)$ is the T-duality group.

- The $(\mathbf{2}, \mathbf{12})$ is identified as the NS-NS sector where as the $(\mathbf{1}, \mathbf{32})$ is associated with the R-R charges.
- In the Fano plane picture going from NS to NS+RR is going from quaternions to octonions. Suggestive of hidden role of octonions in M-theory?

Superqubits

- We provide a supersymmetric generalisation of n quantum bits by extending the LOCC entanglement equivalence group $[SU(2)]^n$ to the supergroup $[uOSp(2|1)]^n$ and the SLOCC equivalence group $[SL(2, C)]^n$ to the supergroup $[OSp(2|1)]^n$.
- We introduce the appropriate supersymmetric generalisations of the conventional entanglement measures for the cases of $n = 2$ and $n = 3$.
- In particular, super-GHZ states are characterised by a non-vanishing superhyperdeterminant.
  L. Borsten, D. Dahanayake, M. J. Duff and W. Rubens, arXiv:0908.0706 [quant-ph].

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