Mergers Involving Black Holes and Neutron Stars in an ADM Landscape

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with input from

Luciano Rezzolla (AEI) and Ed Seidel (NSF)

ADM and Numerical Relativity

- The ADM formulation, with first-order equations and a clear distinction among dynamical, constrained, and gauge variables, was the perfect starting point for numerical relativity.
- ADM provided a framework for

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Kinematical conditions in the construction of spacetime

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ADM Equations

$$\partial_t K_{ij} = -D_i D_j \alpha + \alpha(R_{ij} - 2K_{ik}K^{kj} + KK_{ij}) \\ -8\pi\alpha(R_{ij} - \frac{1}{2}\gamma_{ij}(S - e)) + \mathcal{L}_{\boldsymbol{\beta}} K_{ij}$$

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \mathcal{L}_{\boldsymbol{\beta}} \gamma_{ij}$$

$$R + K^2 - K_{ij}K^{ij} = 16\pi e$$

$$D_j K^j{}_i - D_i K = 8\pi j_i$$

Stable Numerical Integration

- Early numerical simulations typically developed instabilities, crashing after a short time.
- Because computers were limited, it was not clear for a long time whether the problem was caused by poor resolution, close boundaries, singularities, horizons, the formulation of the equations themselves -- or all of the above!
- Finally the AEI group, especially Alcubierre, showed that a key problem was the ADM split between "evolution" and "constraint" equations. The system is only weakly hyperbolic.
- The BSSN (Baumgarte-Shapiro-Sasaki-Nakamura)

BSSN Variables

$$\phi = \frac{1}{12}\ln(\det(\gamma_{ij})) = \frac{1}{12}\ln(\gamma),$$

$$\tilde{\gamma}_{ij} = e^{-4\phi}\gamma_{ij},$$

$$K = \gamma^{ij}K_{ij},$$

$$\tilde{A}_{ij} = e^{-4\phi}(K_{ij} - \frac{1}{3}\gamma_{ij}K),$$

$$\Gamma^i = \gamma^{jk}\Gamma^i_{jk}$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk}$$

ϕ: conformal factor

$\tilde{\gamma}_{ij}$: conformal 3-metric

K: trace of extrinsic curvature

$\tilde{A}_{ij}$: trace-free conformal extrinsic curvature

$\tilde{\Gamma}^i$:“Gammas”

are our new evolution variables

ADM equations a la BSSN

$$\mathcal{D}_t \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} \ , \quad \text{where } \mathcal{D}_t \equiv \partial_t - \mathcal{L}_\beta$$

$$\mathcal{D}_t \phi = -\frac{1}{6}\alpha K \ ,$$

$$\mathcal{D}_t \tilde{A}_{ij} = e^{-4\phi}\left[-\nabla_i \nabla_j \alpha + \alpha \left(R_{ij} - S_{ij}\right)\right]^{\mathrm{TF}} + \alpha\left(K \tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j\right) \ ,$$

$$\mathcal{D}_t K = -\gamma^{ij}\nabla_i \nabla_j \alpha + \alpha\left[\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + \frac{1}{2}\left(\rho + S\right)\right] \ ,$$

$$\mathcal{D}_t \tilde{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha\left(\tilde{\Gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K - \tilde{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi\right)$$
$$-\partial_j\left(\beta^l \partial_l \tilde{\gamma}^{ij} - 2\tilde{\gamma}^{m(j}\partial_m \beta^{i)} + \frac{2}{3}\tilde{\gamma}^{ij}\partial_l \beta^l\right) \ .$$

The ADM Landscape for NumRel

- Including the BSSN reformulation of the equations, the ADM framework is the starting point for 90% of numerical relativity simulations.
- ADM notation and conceptualization remain the language of numerical relativity: lapse ***N***, shift β^i, slice, extrinsic 3-curvature ***K***, intrinsic 3-curvature ***R***, 3-metric and 3-momentum ***g*** and *π*.
- Success in numerical relativity needed

Numerical Relativity and the AEI

- AEI founded 1995, Ed Seidel arrived in 1996. Group members over the years included Masso (Cactus), Brandt & Bruegmann (invented punctures for initial value problem), Walker (Cactus), Alcubierre (numerical theory), Allen (Cactus), Campanelli and Lousto (Lazarus), Koppitz (first evolution with fixed punctures), Baker, Pollney, Ott, ... Research focus: methods, tools, binary BH problem.
- "Breakthroughs": Pretorius (harmonic formulation), then within ADM/BSSN the use of *moving* punctures 2007: Baker and Koppitz worked with Centrella at GSFC, simultaneously Campanelli and Lousto at UTB. Final piece of a complex puzzle fell into place -- efficient stable evolutions now routine.
- AEI group now led by Luciano Rezzolla. Research focus: exploitation -- BBH, BNS, fluids, MHD, providing waveforms for GW searches. Tools: Cactus/Carpet/Whisky code.

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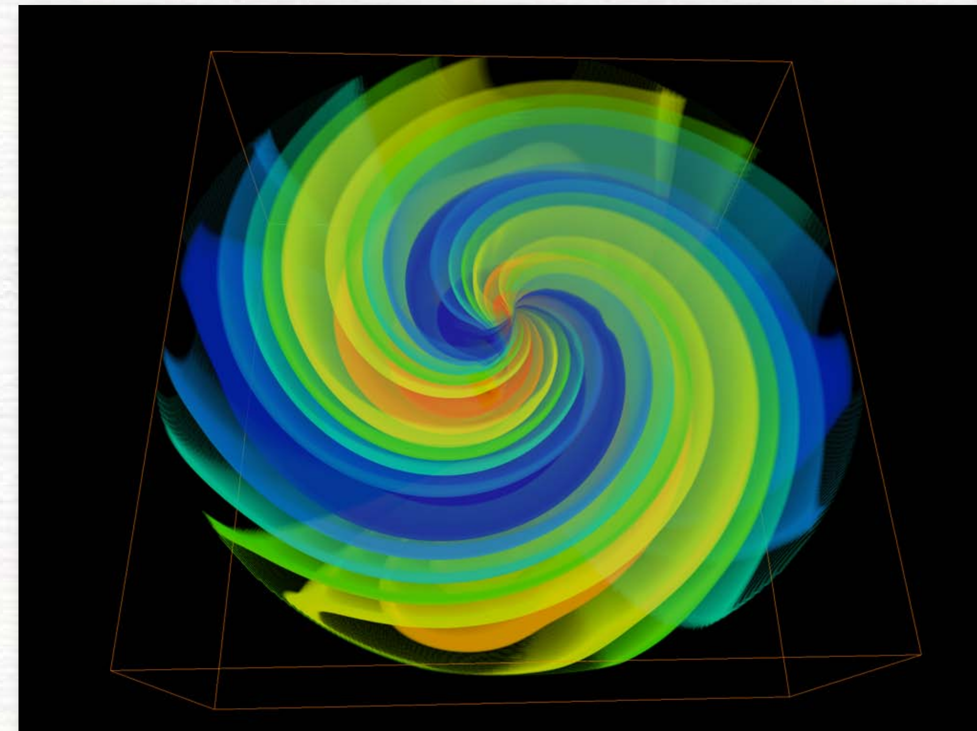
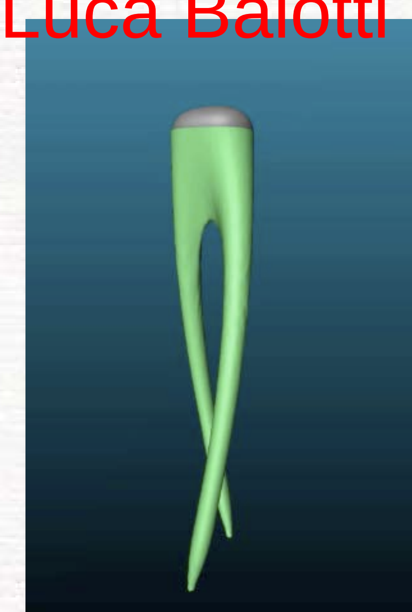
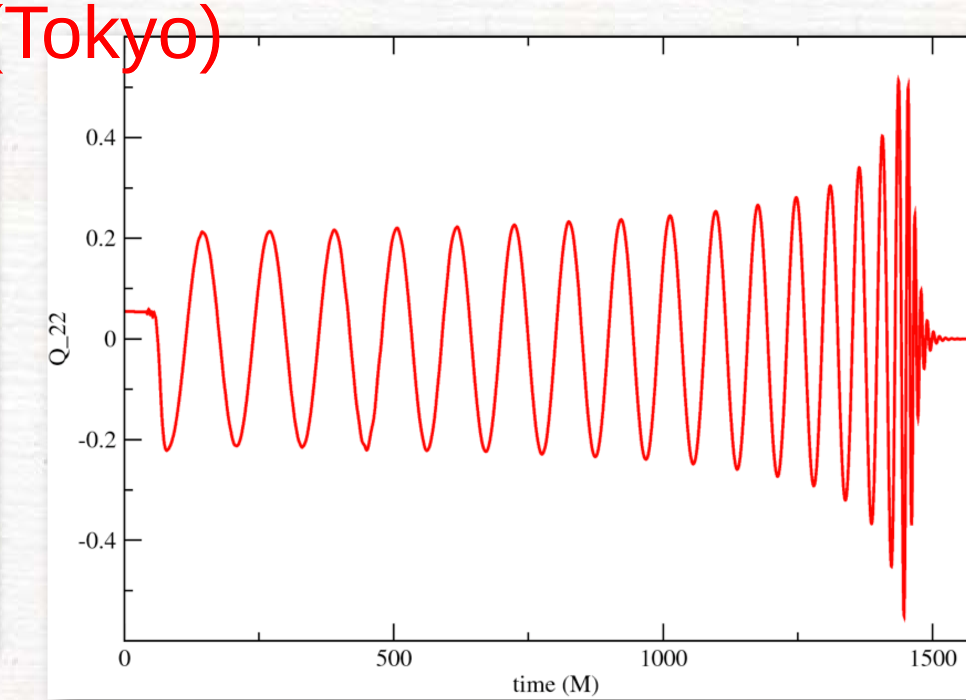


Binary black holes





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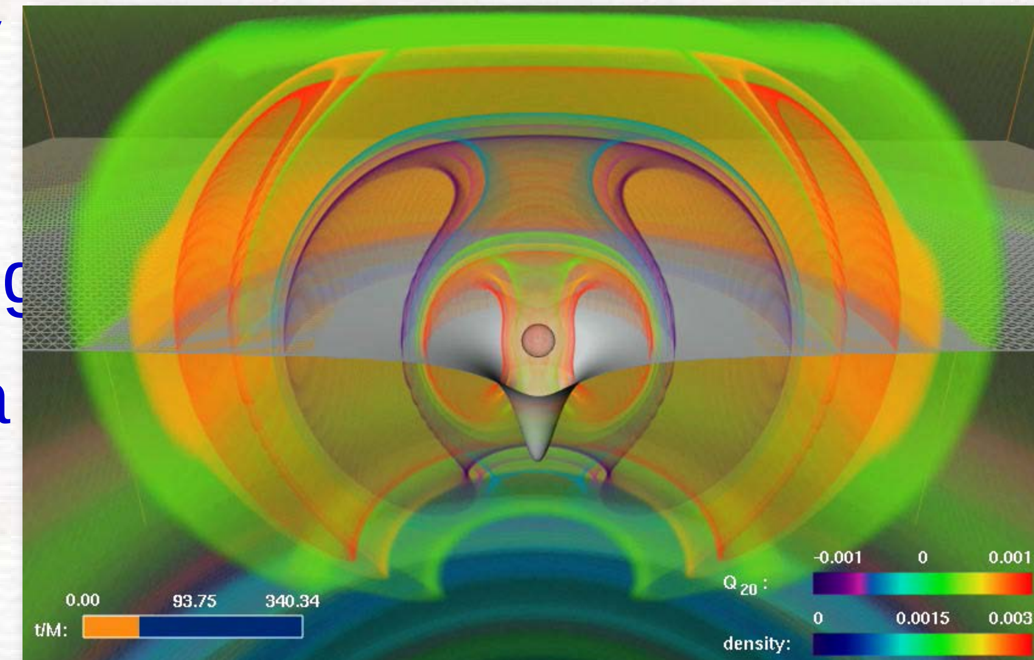
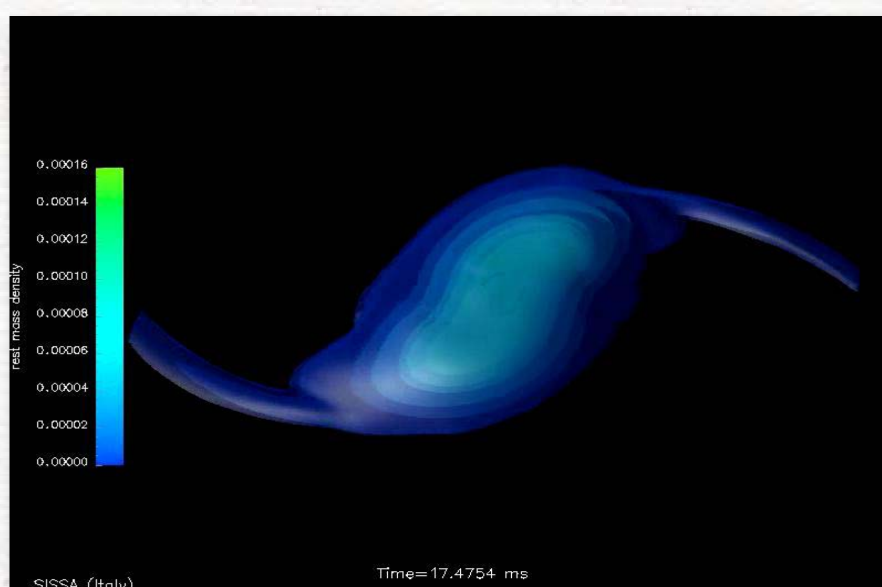
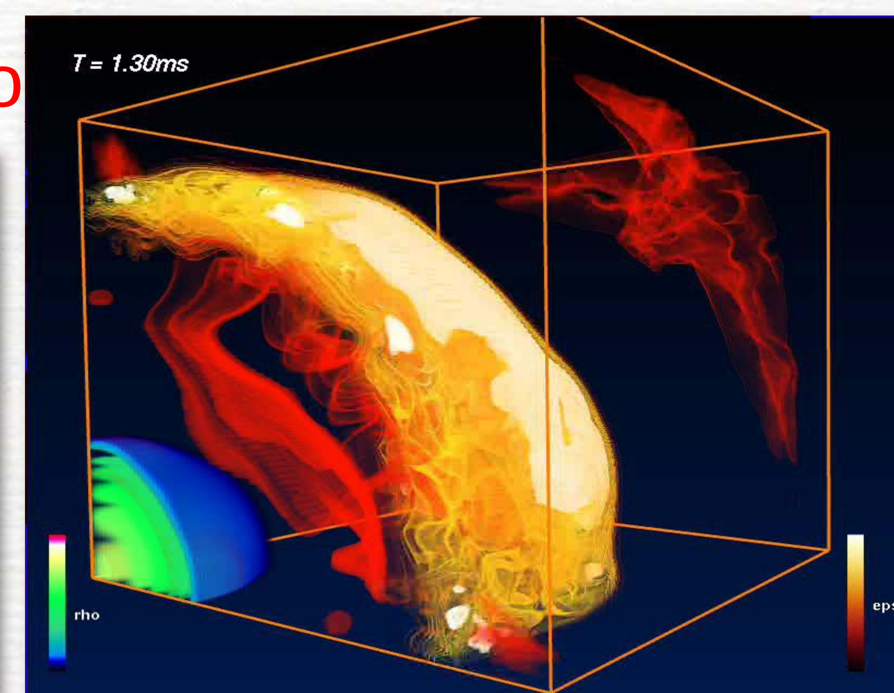


Isolated NSs, perturbation





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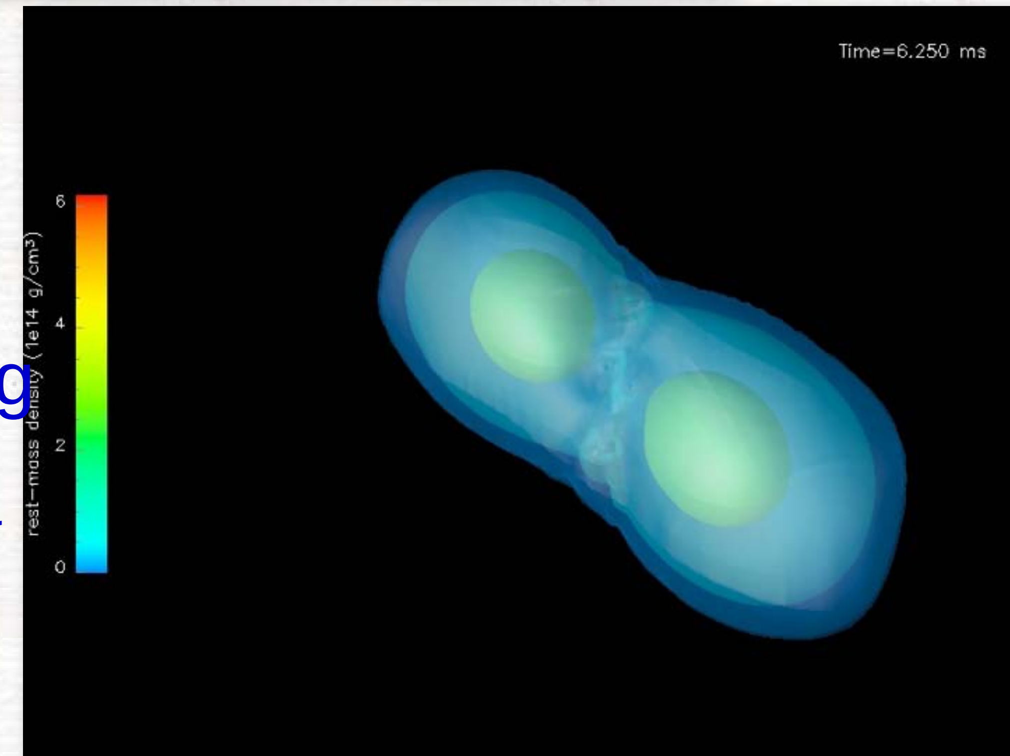
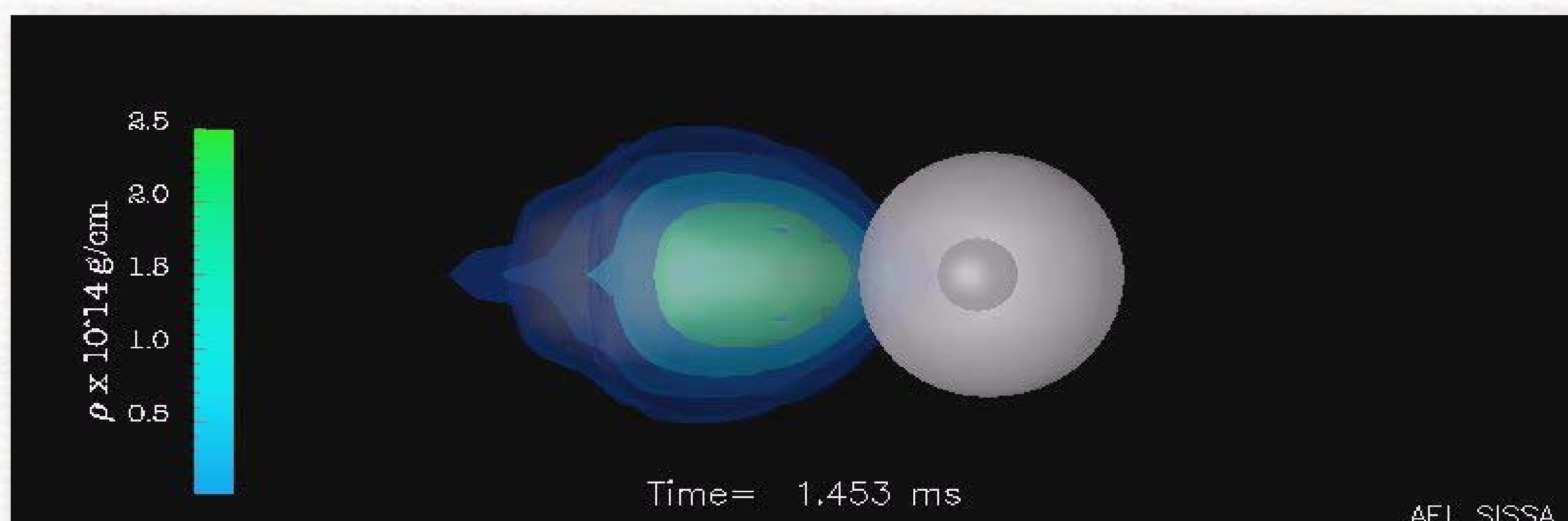
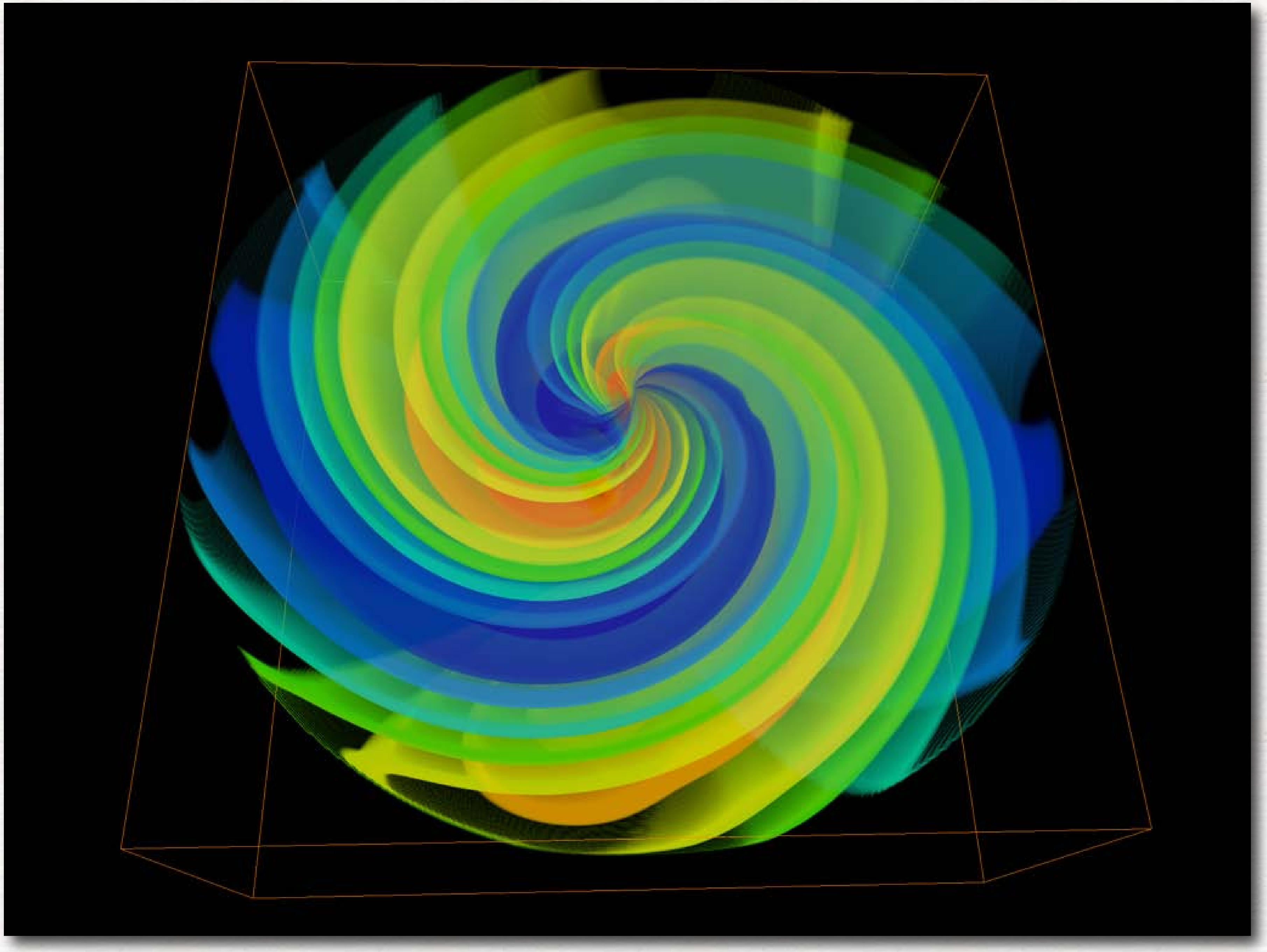




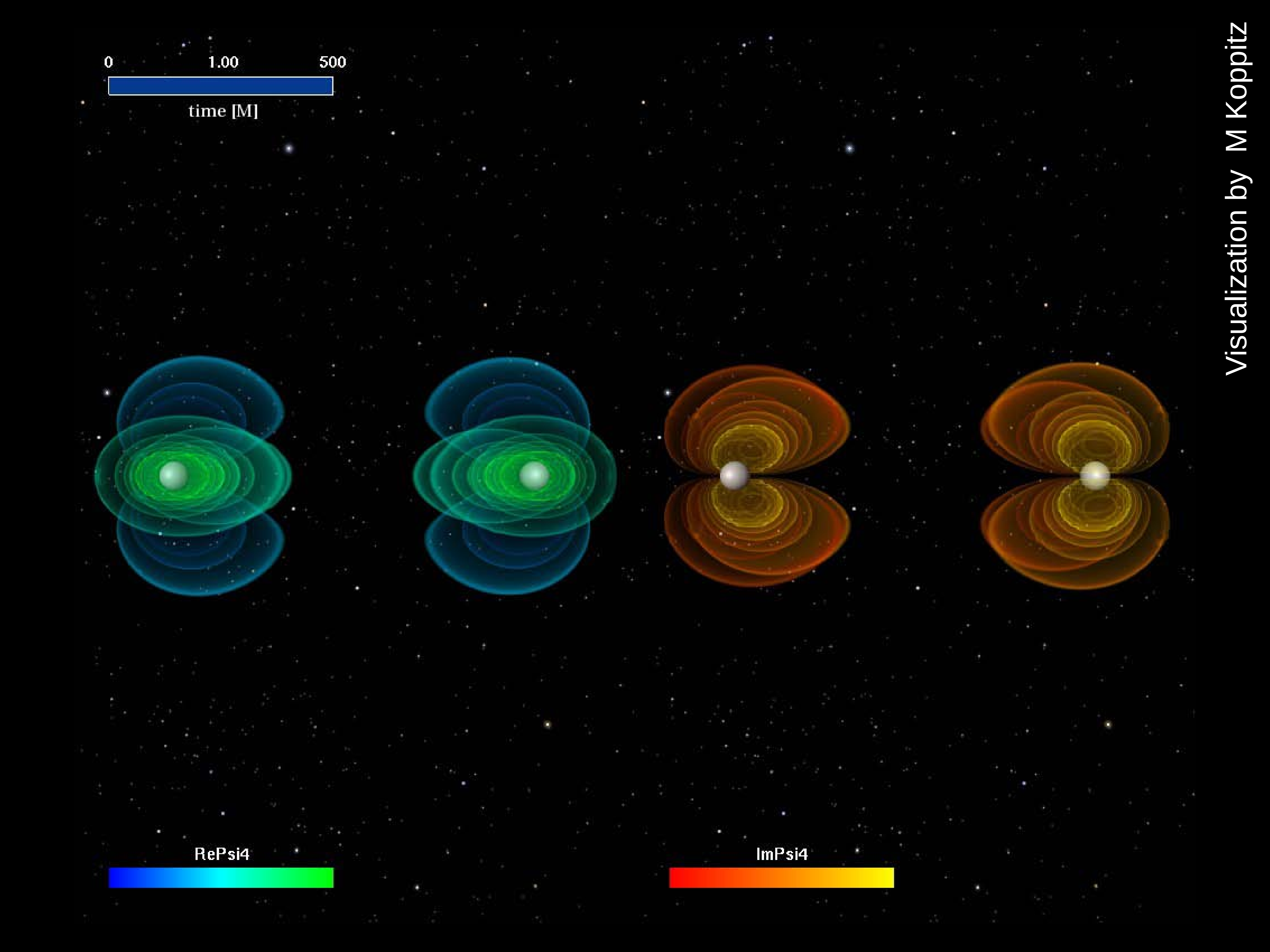
BH-NS

NS-NS binaries

Binary black holes



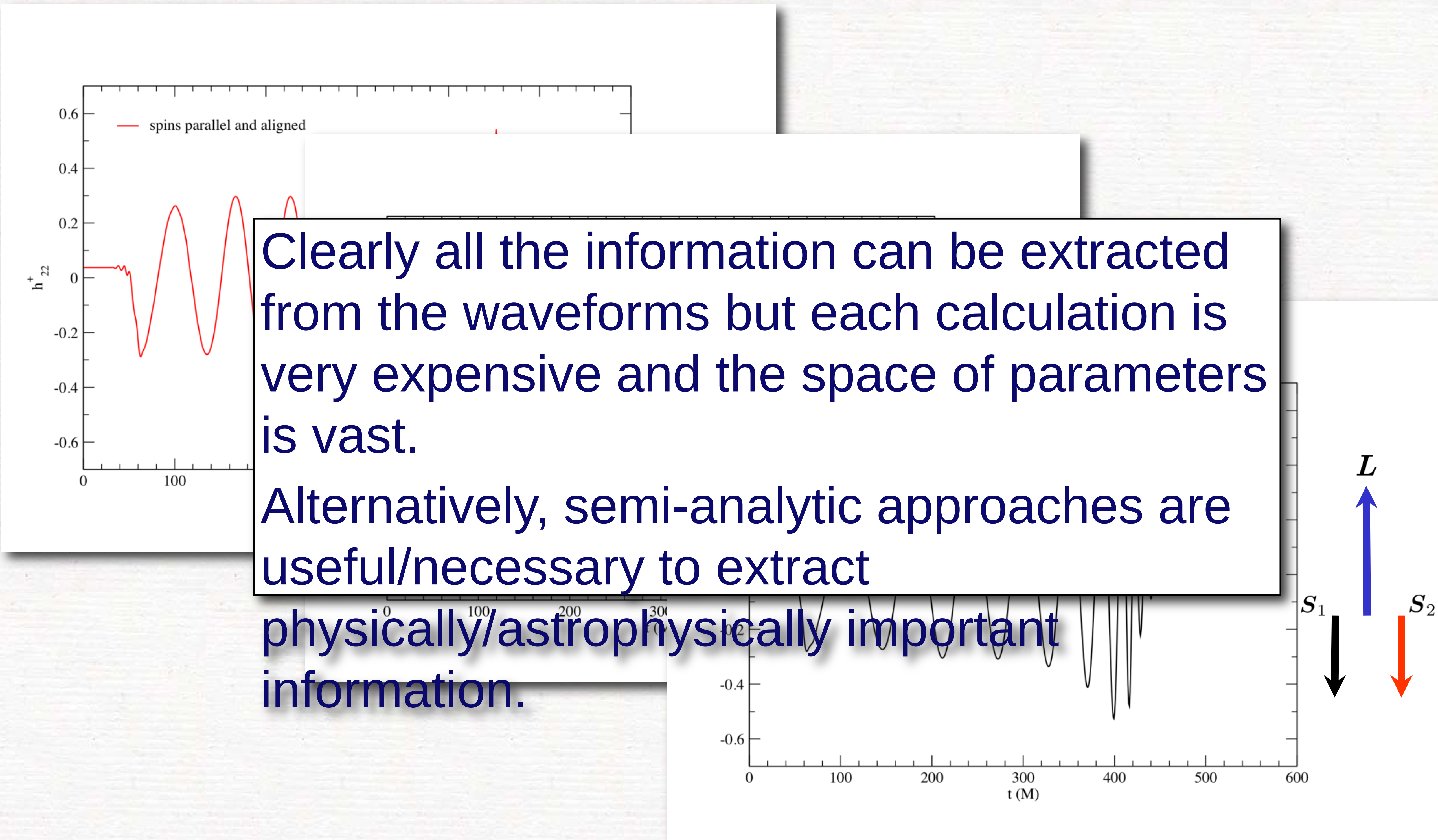
Koppitz et al. PRL 2007
Pollney et al., PRD 2007
LR et al, 2007, ApJ
LR et al, 2008 ApJL
LR et al, 2009 PRD
Barausse, LR, ApJL 2009


0
1.00
500
time [M]
RePsi4
ImPsi4
Visualization by M Koppitz

Gravitational Wave Searches

- Numerical waveform predictions are now good enough to improve LIGO-VIRGO data analysis.
- High-mass searches (>30 M_Γ) need numerical waveforms for good sensitivity.
- NINJA project is bringing spinning binary simulations into searches.
- AEI group very active in NINJA: Krishnan, Santamaria, Ajith, Pollney, ...

BBH Simulations Lead to Physics Insight

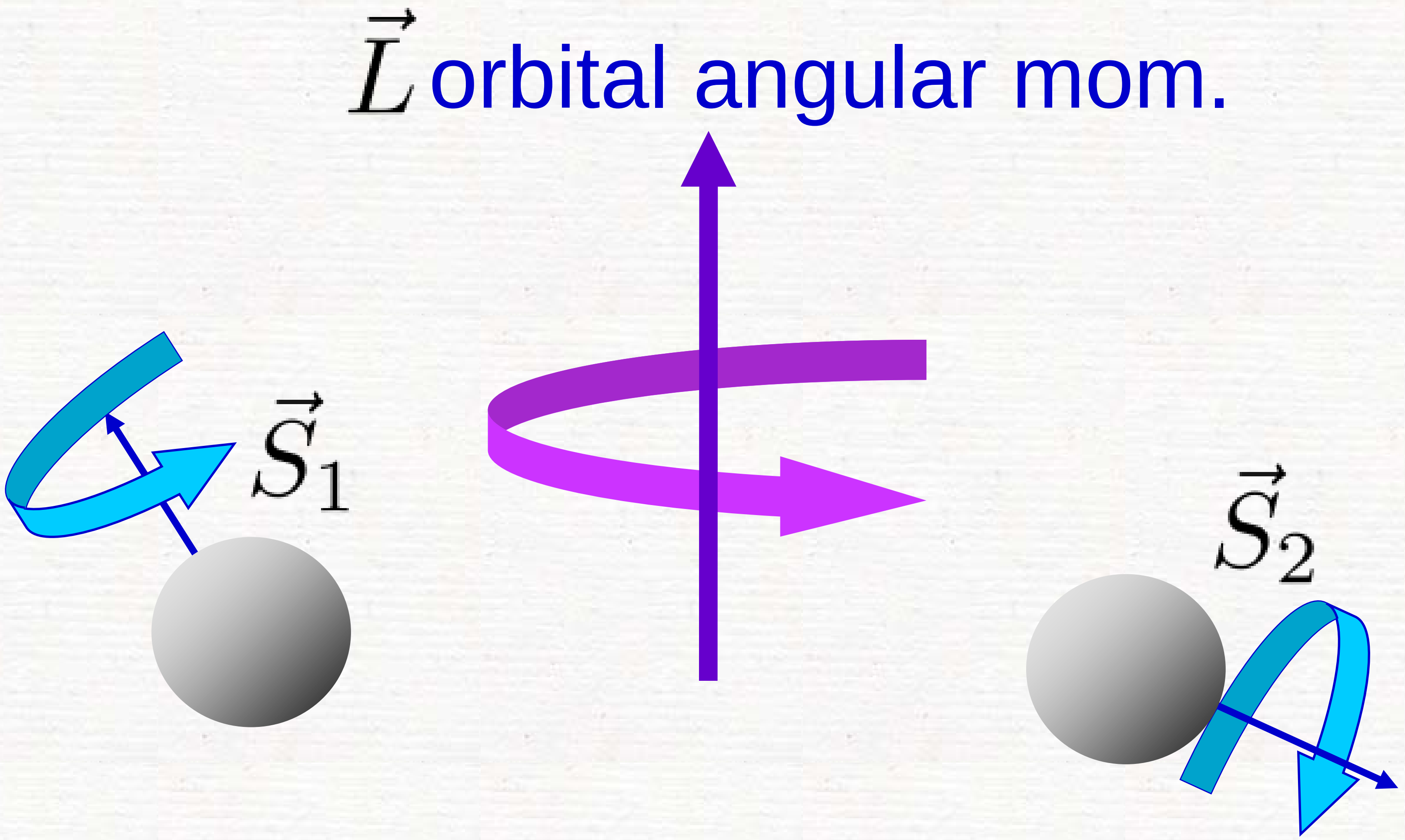


Clearly all the information can be extracted from the waveforms but each calculation is very expensive and the space of parameters is vast.

Alternatively, semi-analytic approaches are useful/necessary to extract physically/astrophysically important information.

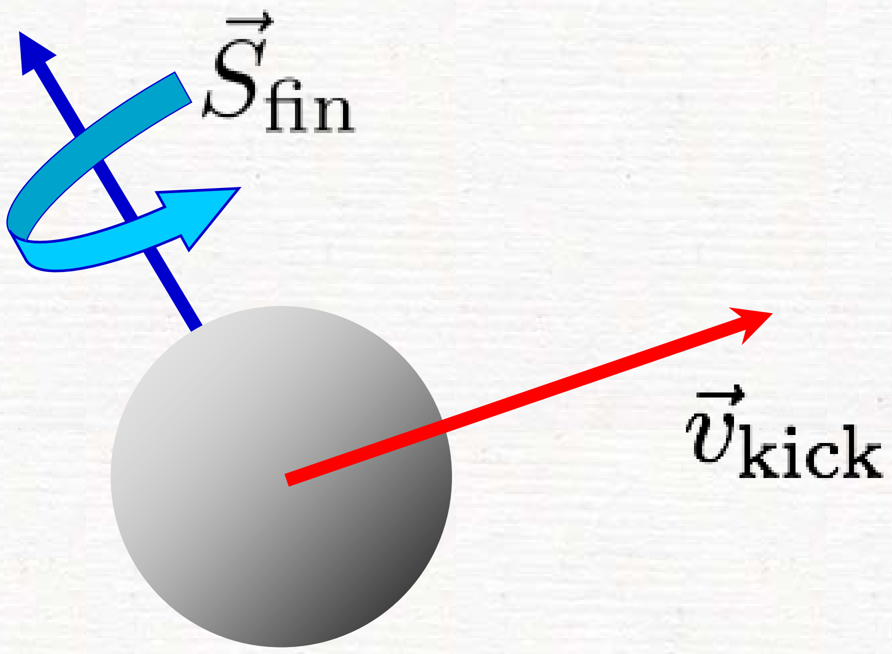
Consider BH binaries as "engines" producing a final single black hole from two distinct initial black holes

Before the merger...



Consider BH binaries as "engines" producing a final single black hole from two distinct initial black holes

After the merger...



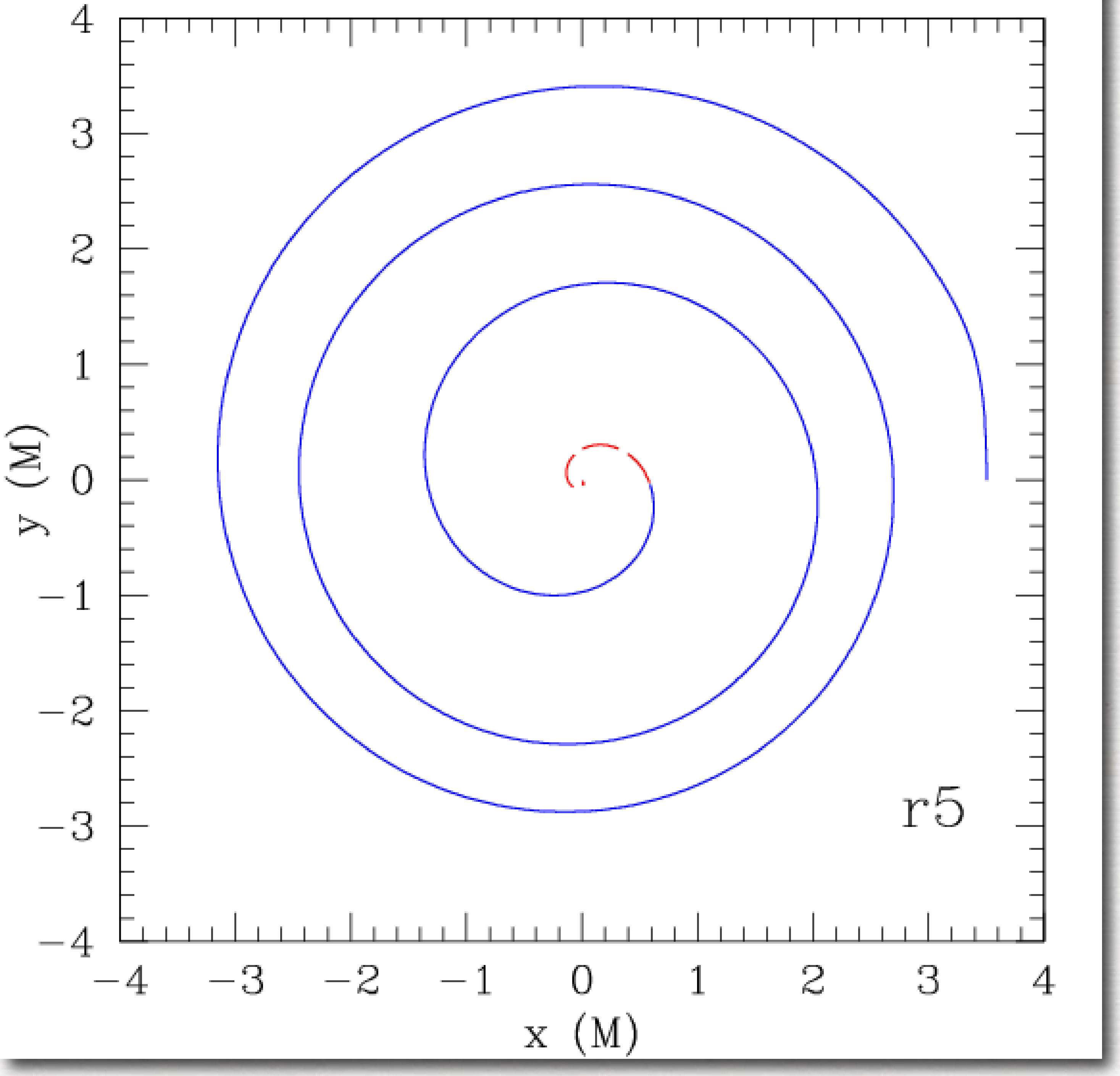
Can we map the initial configuration to a final one without performing a

Campanelli et al, 2006
Campanelli et al, 2007
Baker et al, 2008
Gonzalez et al, 2007
LR et al, 2007
Hermann et al, 2007
LR et al, 2007
Boyle et al, 2007
Marronetti et al, 2007
LR et al, 2007
Boyle et al, 2008
Baker et al, 2008
Lousto et al, 2008
Kesden, 2008
Barausse, LR, 2009

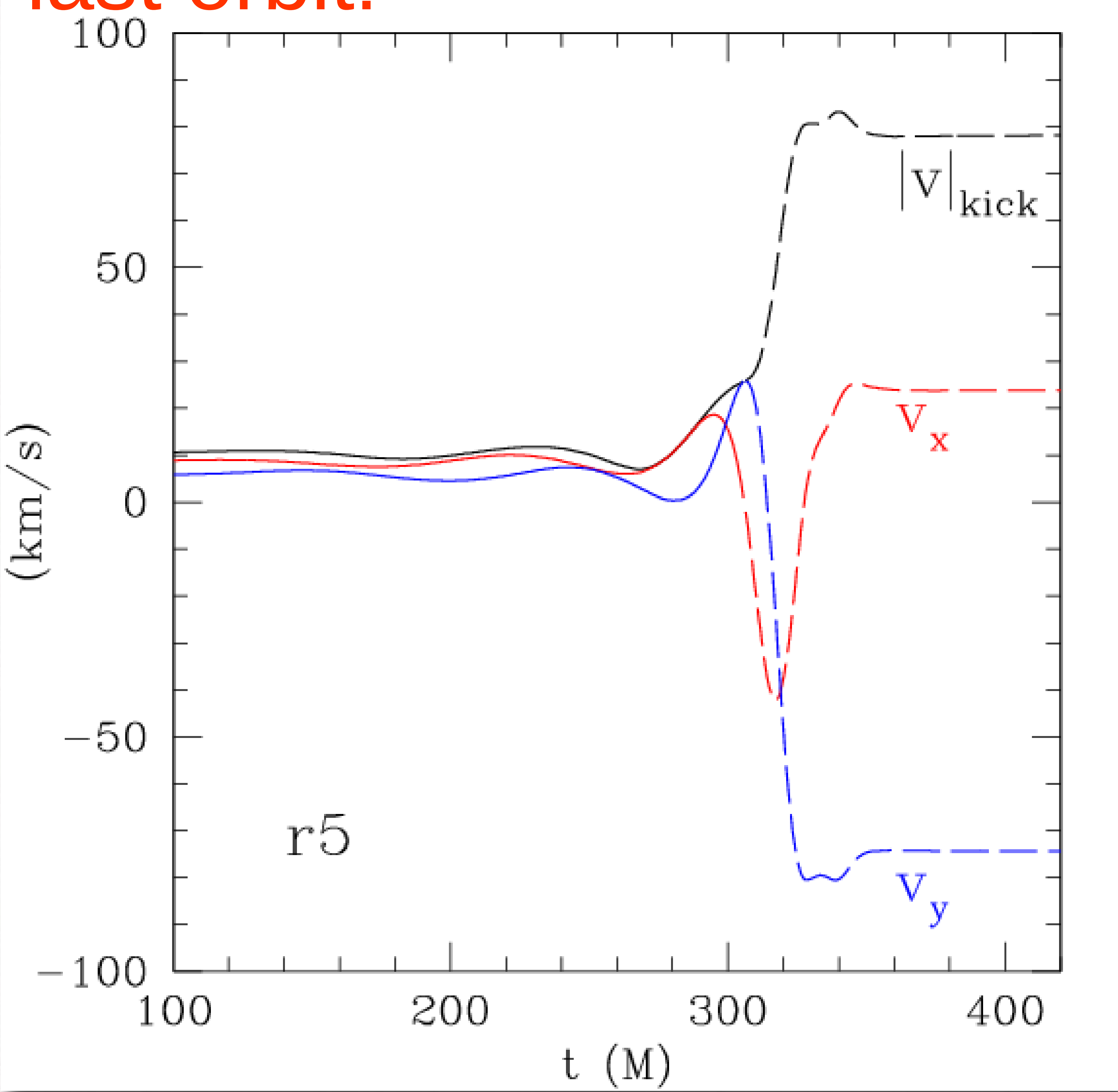
Modelling the final state

- final recoil velocity
- final spin vector

Being sensitive to the asymmetries in the system, the recoil velocity develops very rapidly in the final stages of the inspiral: i.e. during last portion of the last orbit!



Being sensitive to the asymmetries in the system, the recoil velocity develops very rapidly in the final stages of the inspiral: i.e. during last portion of the last orbit!



The details of the processes leading to the recoil are still, in great part, unclear.

Subtle balances in the emission of different QNMs during the ringdown are behind the final kick vector.

Sequences help investigate systematic behaviours in the recoil velocity:, eg r-series: $a_1 \in [-0.584, 0.584], a_2 = 0.584$

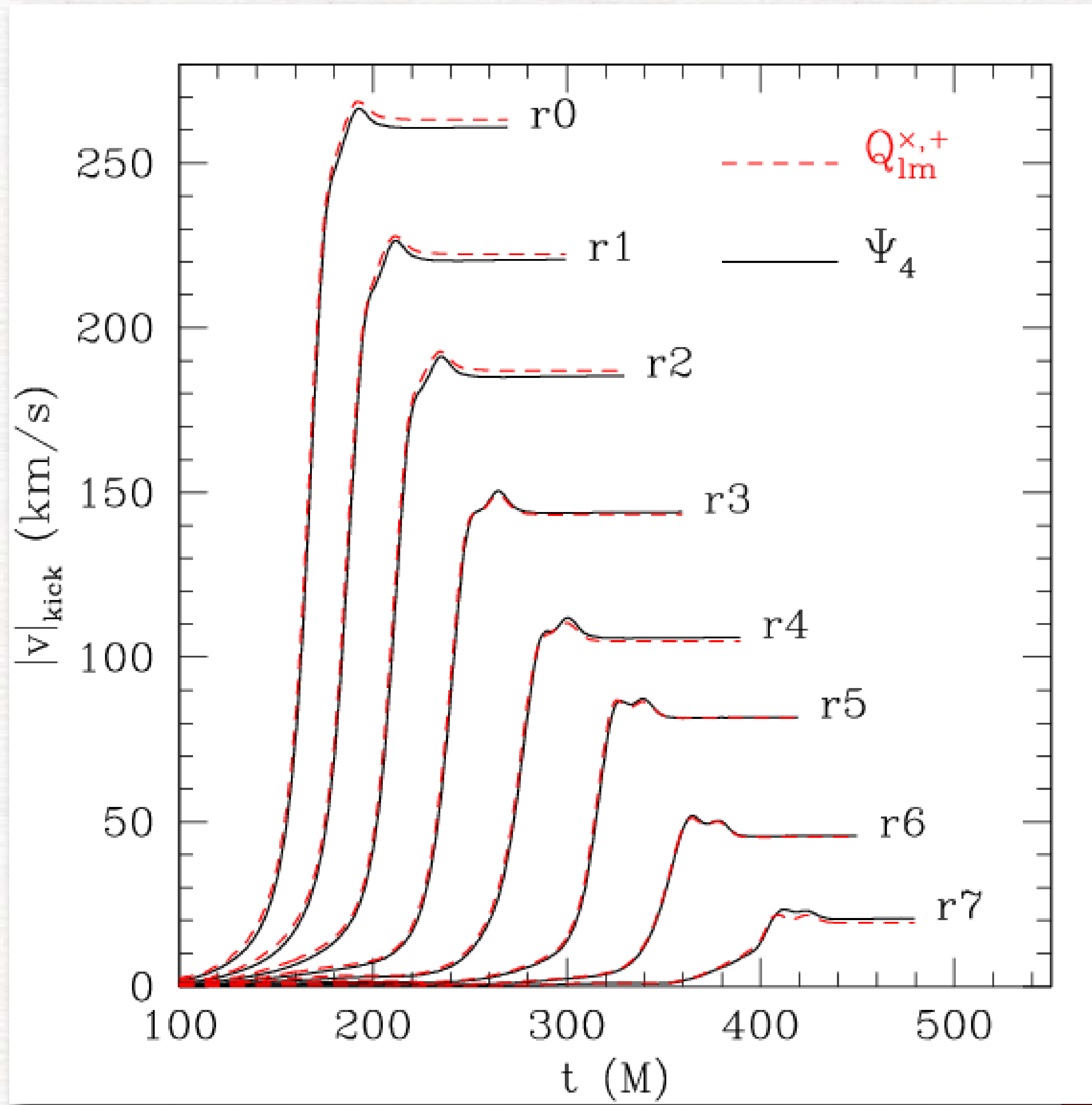
r0: □□ (a_1/a_2=-4/4)

r2: □□ (a_1/a_2=-2/4)

r4: □. (a_1/a_2=-0/4)

r6: □□ (a_1/a_2=2/4)

r8: □□(a_1/a_2=4/4)



What we know (now) of the kick

$$\boldsymbol{v}_{\text{kick}} = v_m\, \boldsymbol{e}_1 + v_\perp (\cos(\xi)\, \boldsymbol{e}_1 + \sin(\xi)\, \boldsymbol{e}_2) + v_\| \, \boldsymbol{e}_3,$$

where

$$v_m = A\nu^2\sqrt{1-4\nu}(1+B\nu),$$

$$v_\perp = c_1 \frac{\nu^2}{1+q}\left(a_2^\| - q a_1^\|\right) + c_2\left((a_2^\|)^2 - q^2 (a_1^\|)^2\right),$$

$$v_\| = \frac{K\nu^3}{(1+q)}\left[q a_1^\perp \cos(\phi_1 - \Phi_1) - a_2^\perp \cos(\phi_2 - \Phi_2)\right],$$

mass asymmetry $\lesssim 150\text{km/s}$

spin asymmetry; contribution off the plane $\lesssim 450\text{km/s}$

spin asymmetry; contribution in the plane $\lesssim 3500\text{km/s}$

Modelling the final state

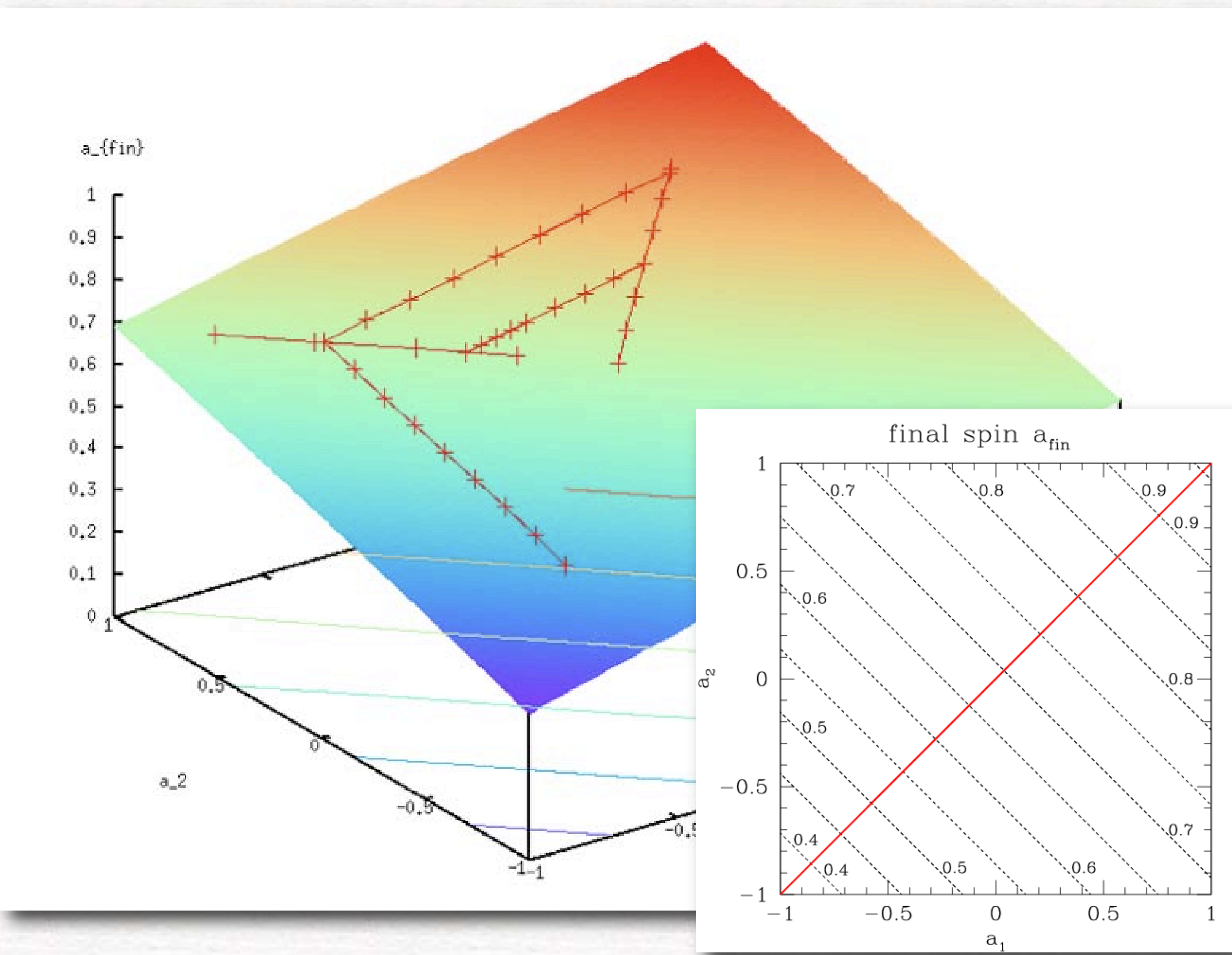
- final recoil velocity
- final spin vector

Equal-mass, unequal-spin binaries

Derive analytical expressions from phenomenological argum
and test them, fit them, to numerical data.

$$a_{\rm fin} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2$$

with $p_0 \simeq 0.6883;\ p_1 \simeq 0.1530;\ p_2 \simeq -0.0088$



- opposite spins same as non spinning
- monotonic behaviour
- final spin increases along the SW-NE diagonal
- minimum and maximum spin $(a_{\rm fin})_{\rm min} \simeq 0.347$

$$(a_{\rm fin})_{\rm max} \simeq 0.959$$

Equal-mass, unequal-spin binaries

$$a_{\rm fin} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2$$

with $p_0 \simeq 0.6883;\ p_1 \simeq 0.1530;\ p_2 \simeq -0.0088$

- angular momentun not radiated: $\lesssim 70\%$

- opposite spins same as non spinning
- monotonic behaviour
- final spin increases along the SW-NE diagonal
- minimum and maximum spin $(a_{\rm fin})_{\rm min} \simeq 0.347$

$$(a_{\rm fin})_{\rm max} \simeq 0.959$$

Equal-mass, unequal-spin binaries

$$a_{\rm fin} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2$$

with $p_0 \simeq 0.6883;\ p_1 \simeq 0.1530;\ p_2 \simeq -0.0088$

- contribution of the initial spins and of the spin-orbit interaction $\lesssim 30\%$

- opposite spins same as non spinning
- monotonic behaviour
- final spin increases along the SW-NE diagonal
- minimum and maximum spin $(a_{\rm fin})_{\rm min} \simeq 0.347$

$$(a_{\rm fin})_{\rm max} \simeq 0.959$$

Equal-mass, unequal-spin binaries

$$a_{\rm fin} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2$$

with $p_0 \simeq 0.6883;\ p_1 \simeq 0.1530;\ p_2 \simeq -0.0088$

- contribution of the initial spins and of the spin-spin interaction: $\lesssim 4\%$

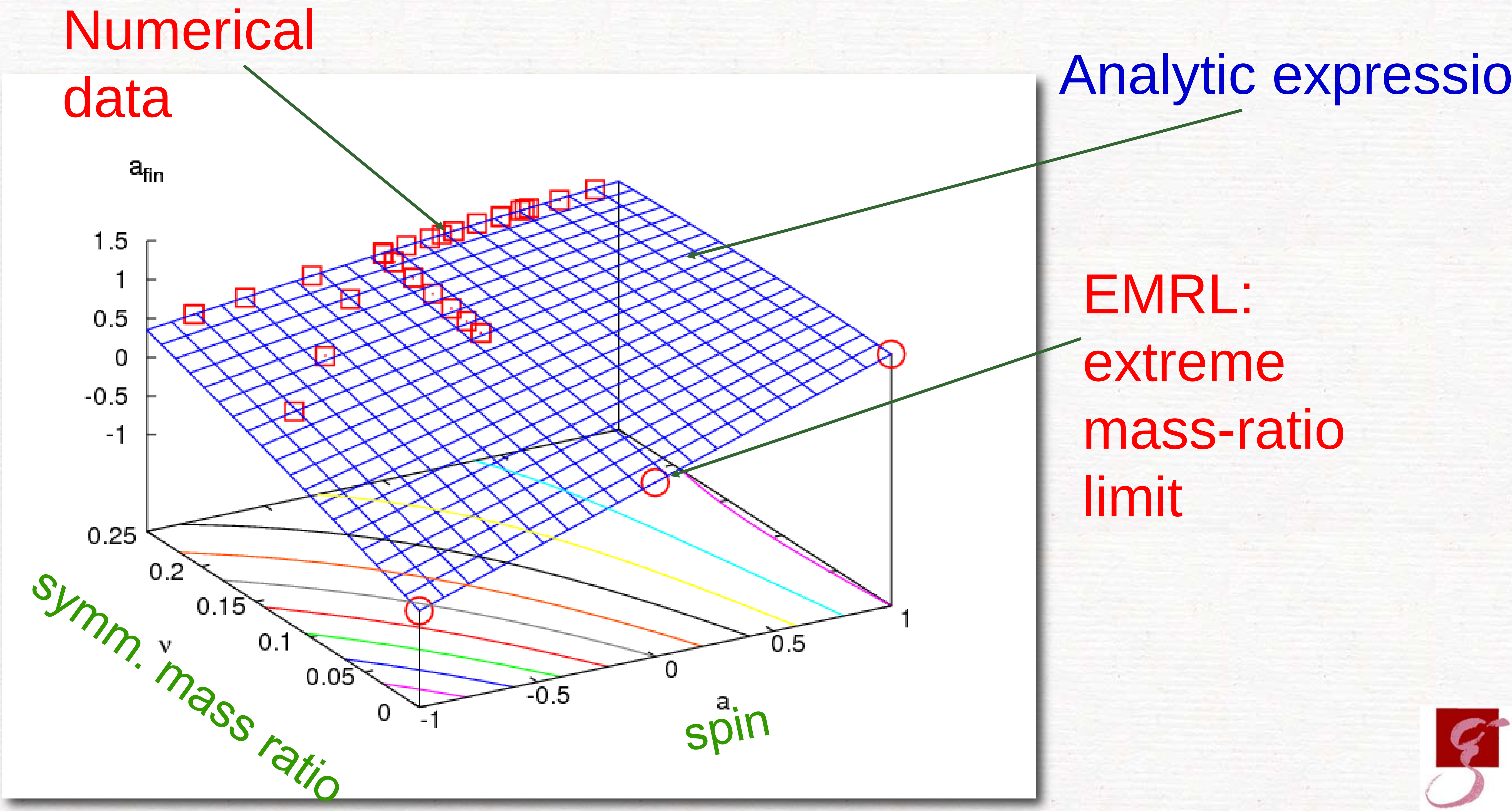
- opposite spins same as non spinning
- monotonic behaviour
- final spin increases along the SW-NE diagonal
- minimum and maximum spin $(a_{\rm fin})_{\rm min} \simeq 0.347$

$$(a_{\rm fin})_{\rm max} \simeq 0.959$$

Unequal-mass, equal-spin binaries

$$\nu = M_1 M_2/(M_1 + M_2)^2$$

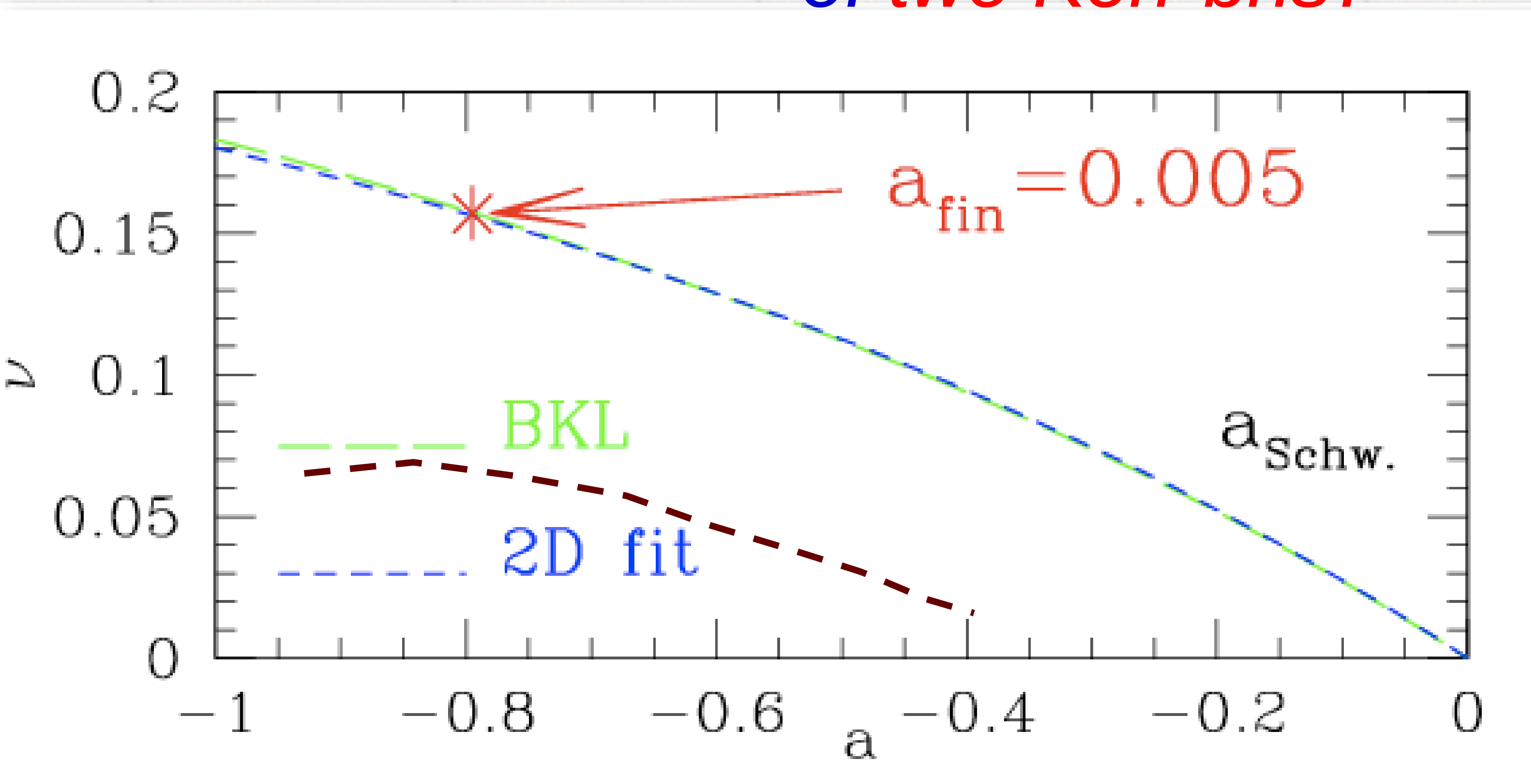
$$a_{\rm fin}(a,\nu) = a + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + t_1 \nu + t_2 \nu^2 + t_3 \nu^3$$



How to produce a Schwarzschild bh...

The analytic expression allows one to answer simple questions like:

Is it possible to produce a Schwarzschild bh from the merger of two Kerr bhs?



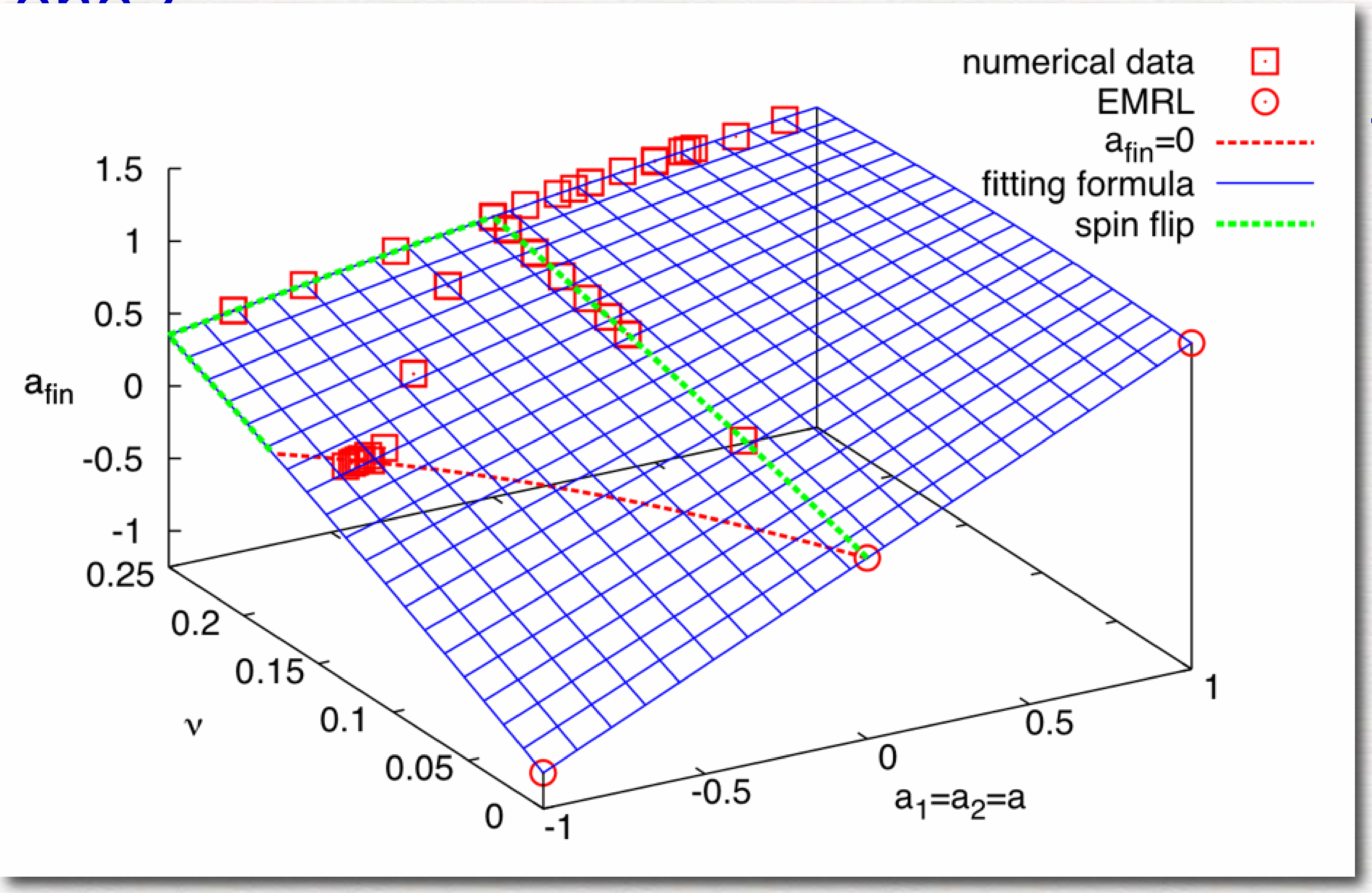
Find solutions for: $a_{\rm fin}(a,\nu) = 0$

Unequal masses and spins antialigned to the orbital ang. mom. are necessary

Isolated Schwarzschild bh likely result of a similar merger!

How to flip the spin...

In other words: under what conditions does the final black hole spin a direction which is opposite to the initial one?



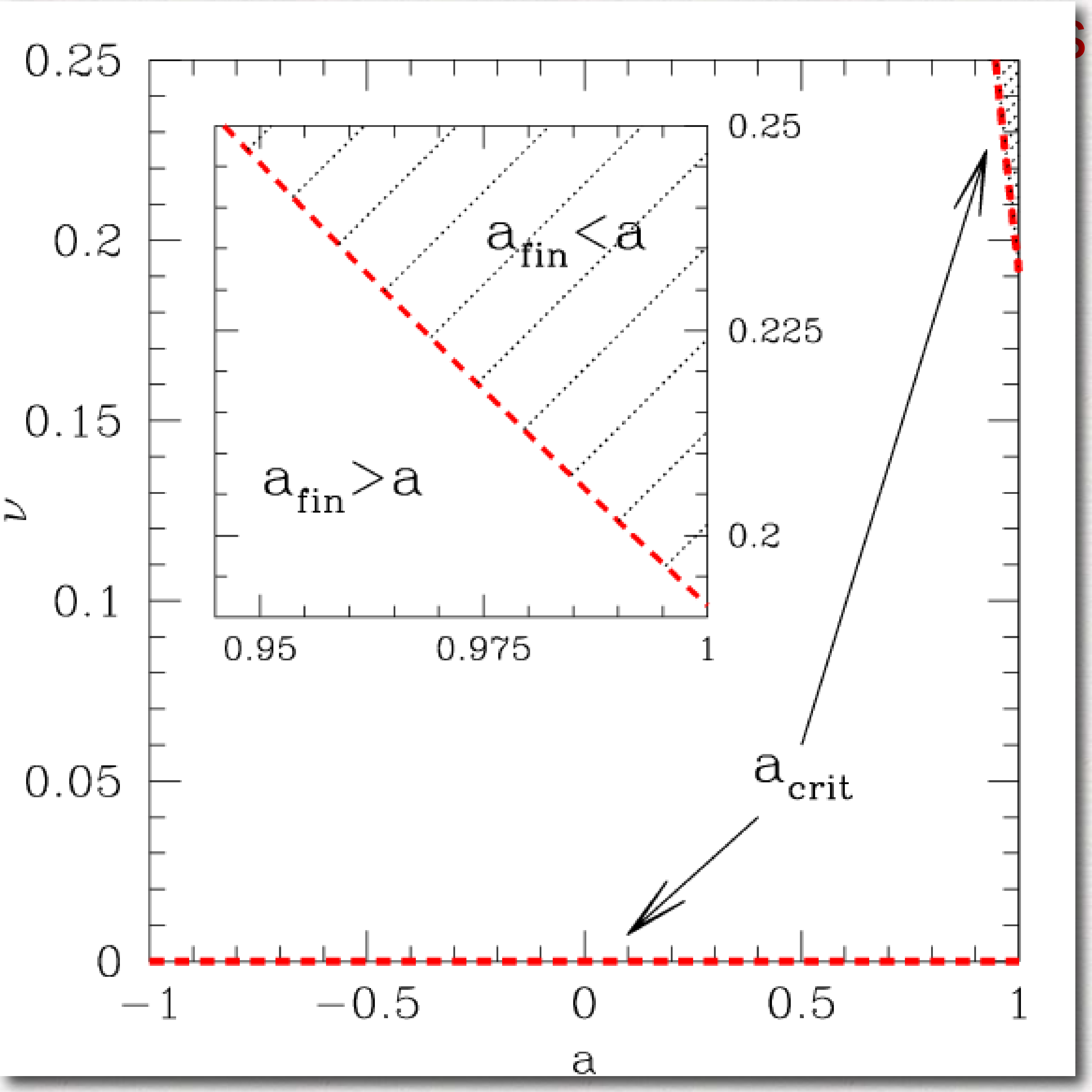
Find solutions for: $a_{\rm fin}(a,\nu)\,a < 0$

Spin-flips are possible if:

- initial spins are anti-aligned with orbital angular mom.
- small spins for small mass ratios
- large spins for comparable masses

Spin-up or spin-down?...

Similarly, another basic question with a simple answer:



spin-up or spin-

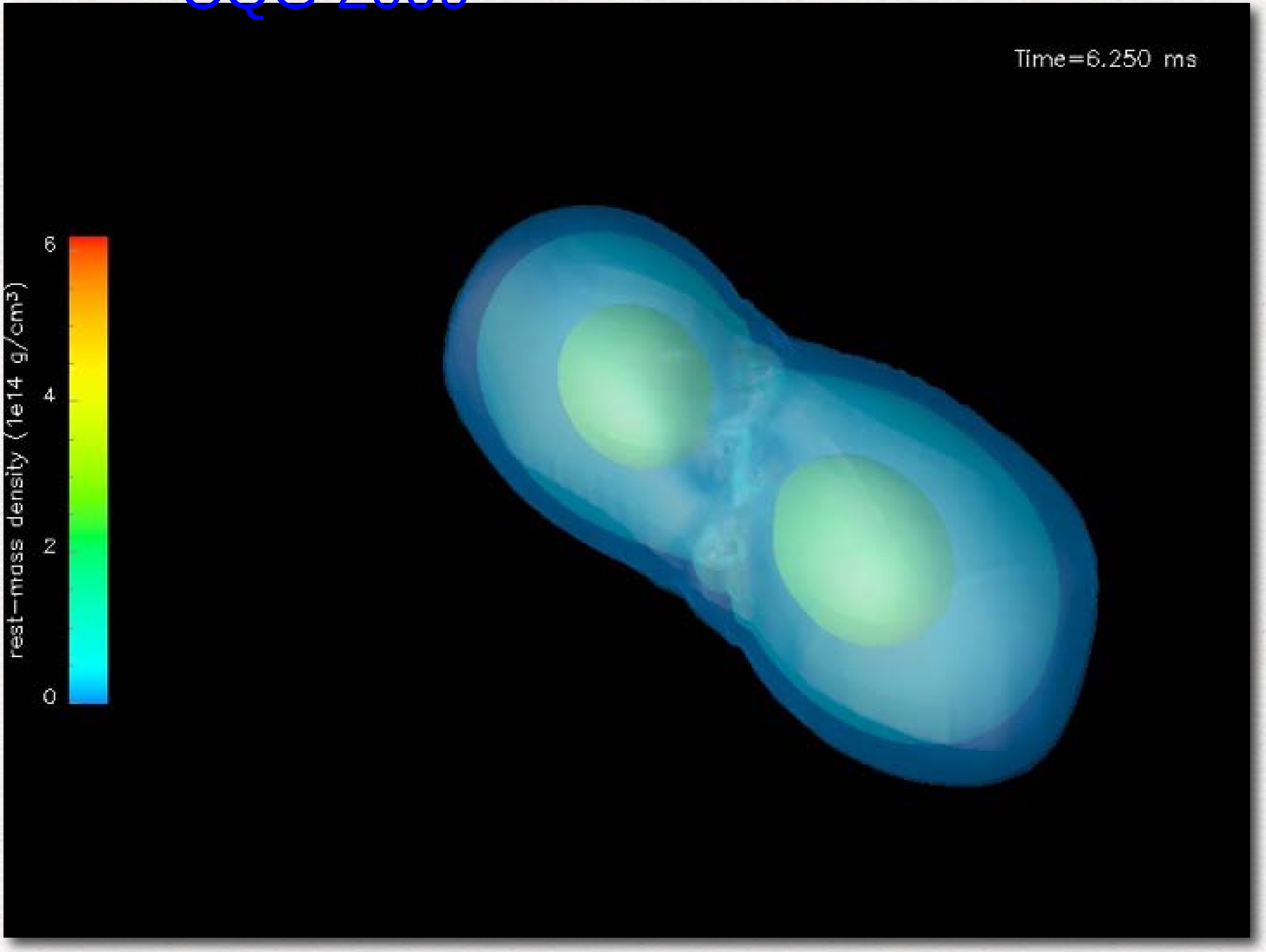
Just find solutions for:

$$a_{\rm fin}(a, \nu) = a$$

Clearly, the merger of aligned BHs statistically, leads to a spin-up. This has impact on modelling the merger of cosmological supermassive BHs

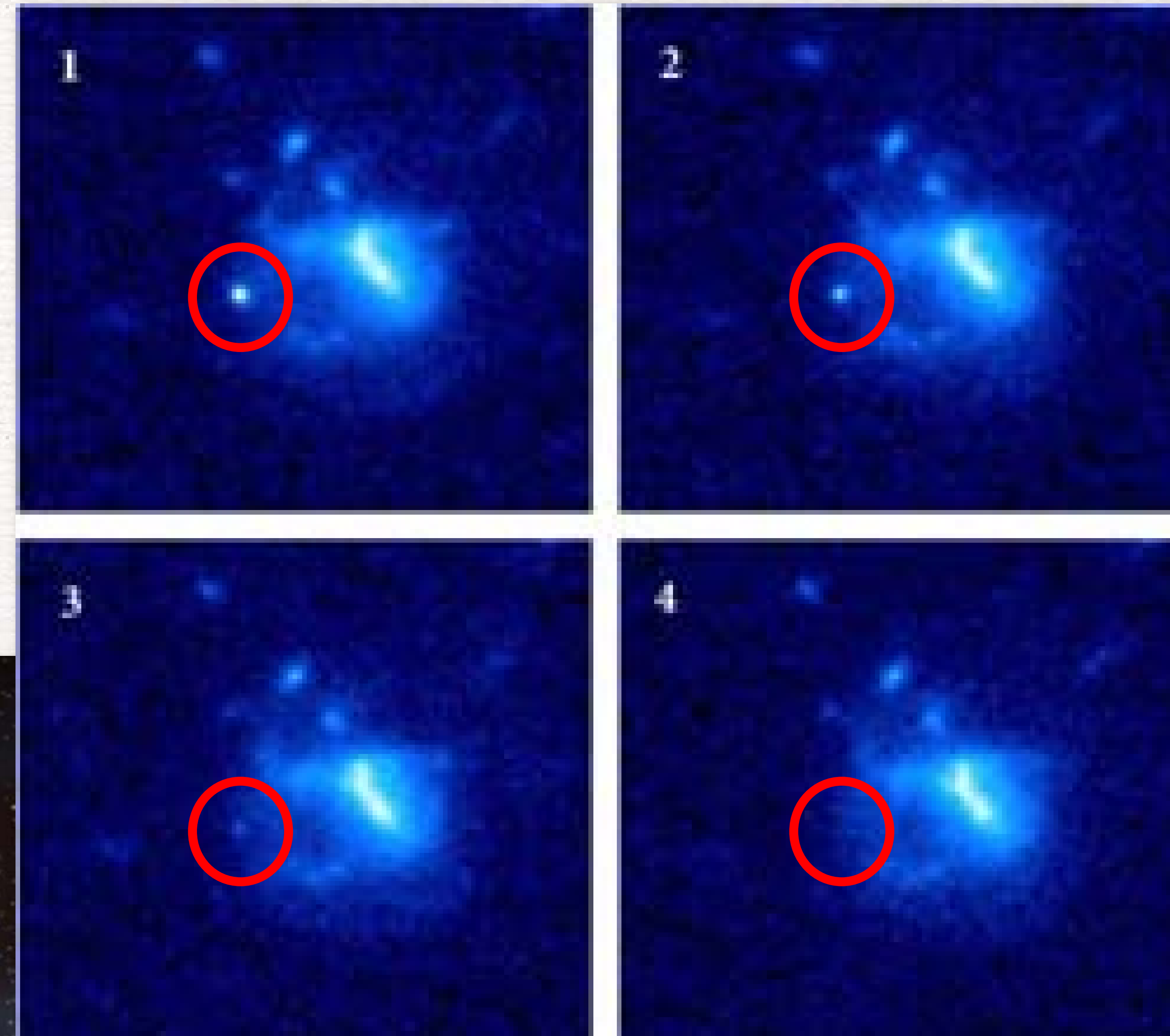
Binary neutron stars

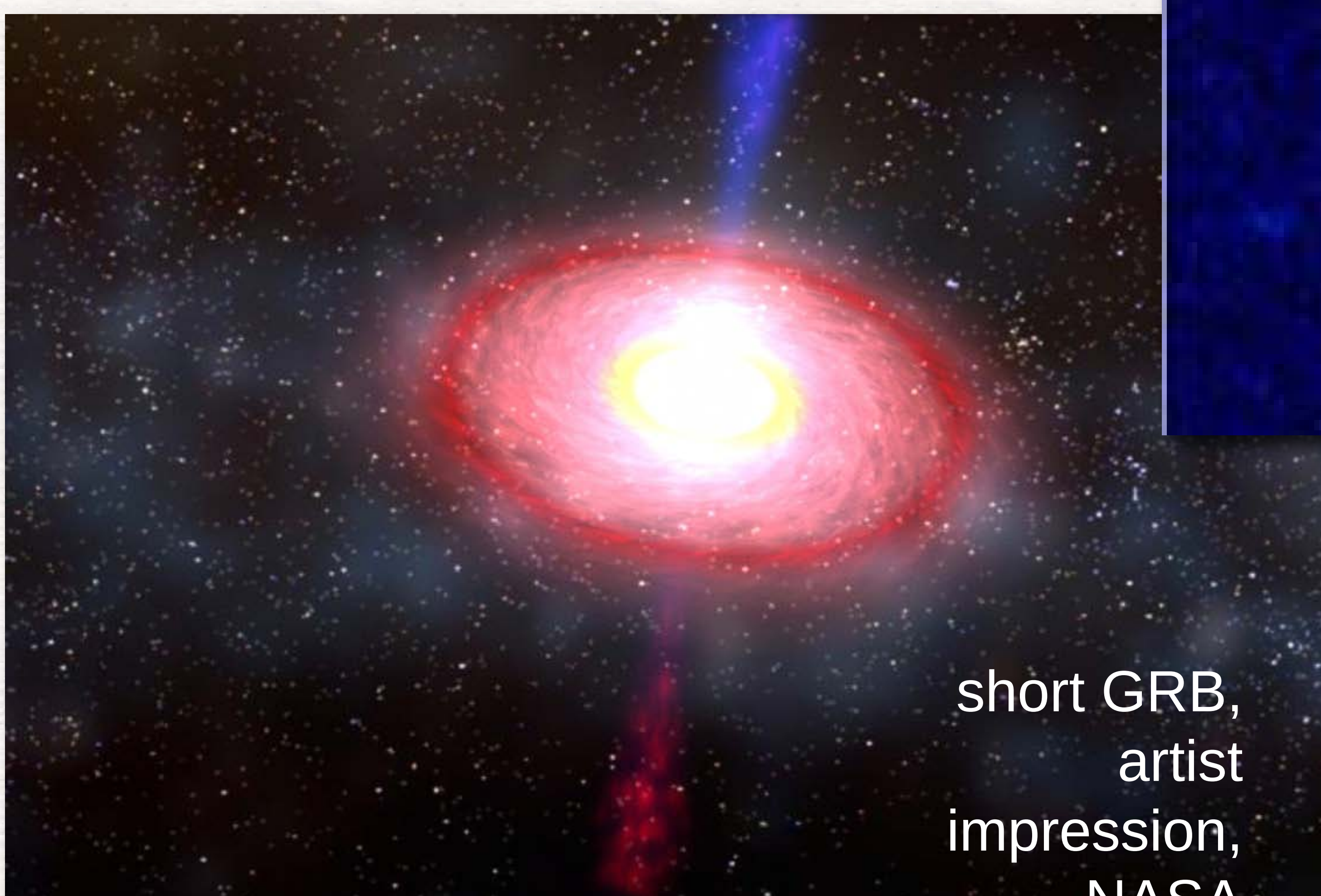
Baiotti, Giacomazzo, Rezzolla, PRD, 2008; CQG 2009



Why study binary neutron stars?

Because they are among the most powerful sources of gravitational waves and could be the Rosetta stone in high-density nuclear physics

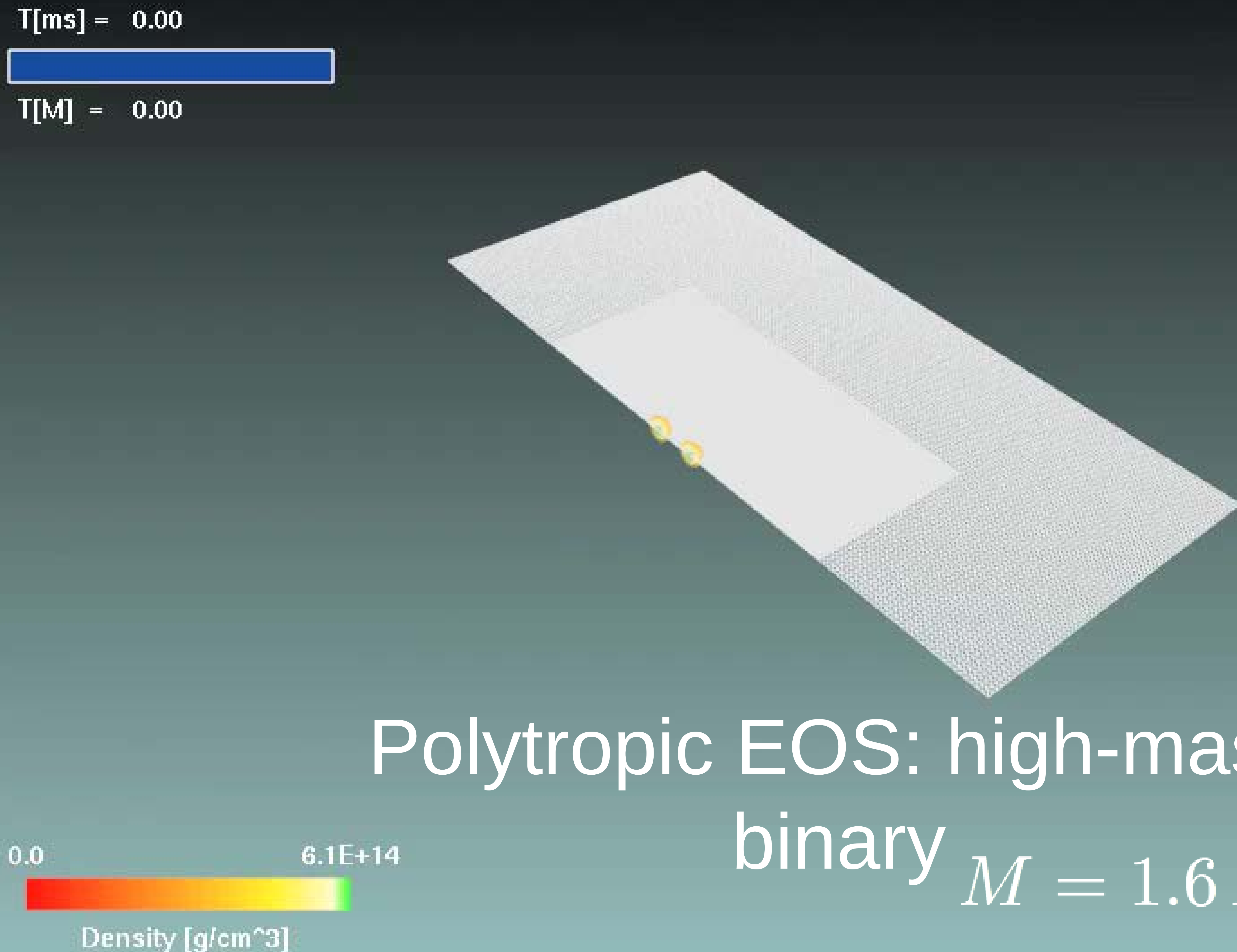




short GRB, artist impression, NASA

Because could be lead to hugely energetic phenomena: short Gamma Ray Bursts (GRBs): 10^{50} erg

Animations: Kaehler, Giacomazzo, Rezzolla

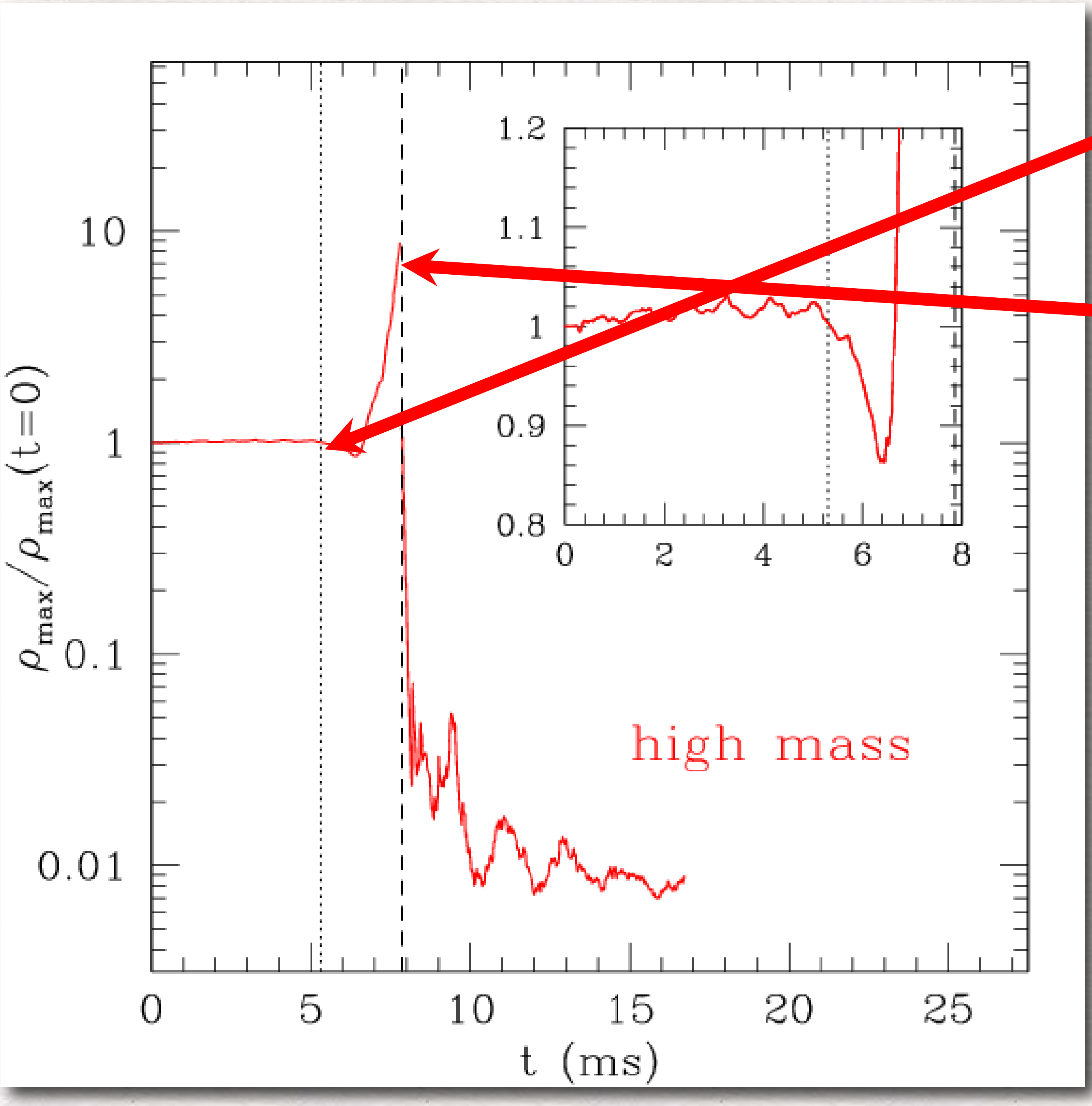


Polytropic EOS: high-mass binary

$M = 1.6\, M_{\odot}$

Matter dynamics

high-mass binary



Merger

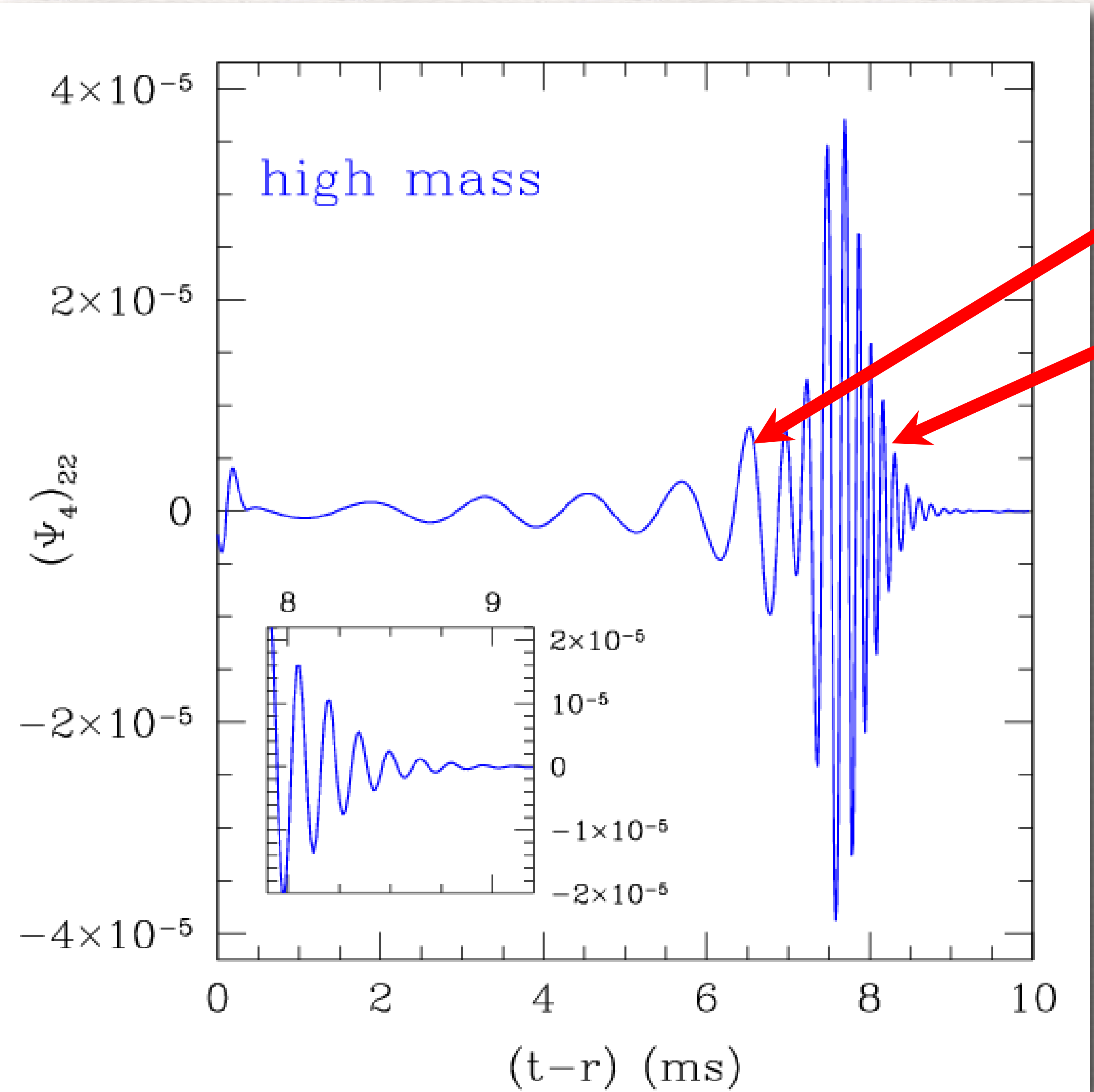
Collapse to BH

soon after the merge the torus is formed and undergoes oscillations

Waveforms: polytropic EOS

high-mass binary

Merger Collapse to BH



first time the full signal from the formation to a bh has been

The behaviour:

“merger → HMNS → BH + torus”

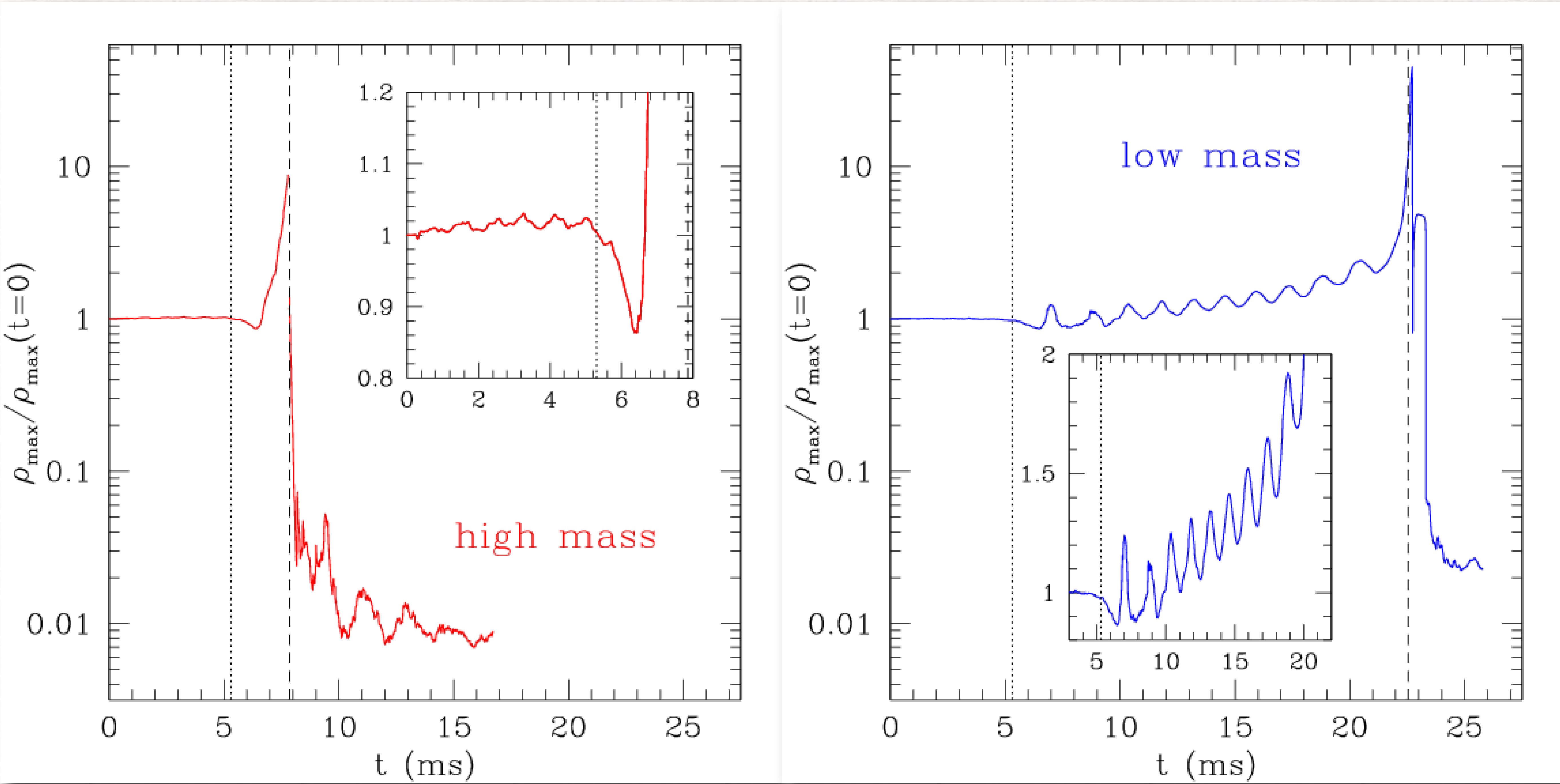
is general but only qualitatively

Quantitative differences are produced by:

- differences in the mass for the same EOS:
  a binary with smaller mass will produce a HMNS which is further away from the stability threshold and will collapse at a later time
- differences in the EOS for the same mass:
  a binary with an EOS allowing for a larger thermal internal energy (ie hotter after merger) will have an increased pressure support and will collapse at a later time

Matter dynamics

high-mass binary | low-mass binary

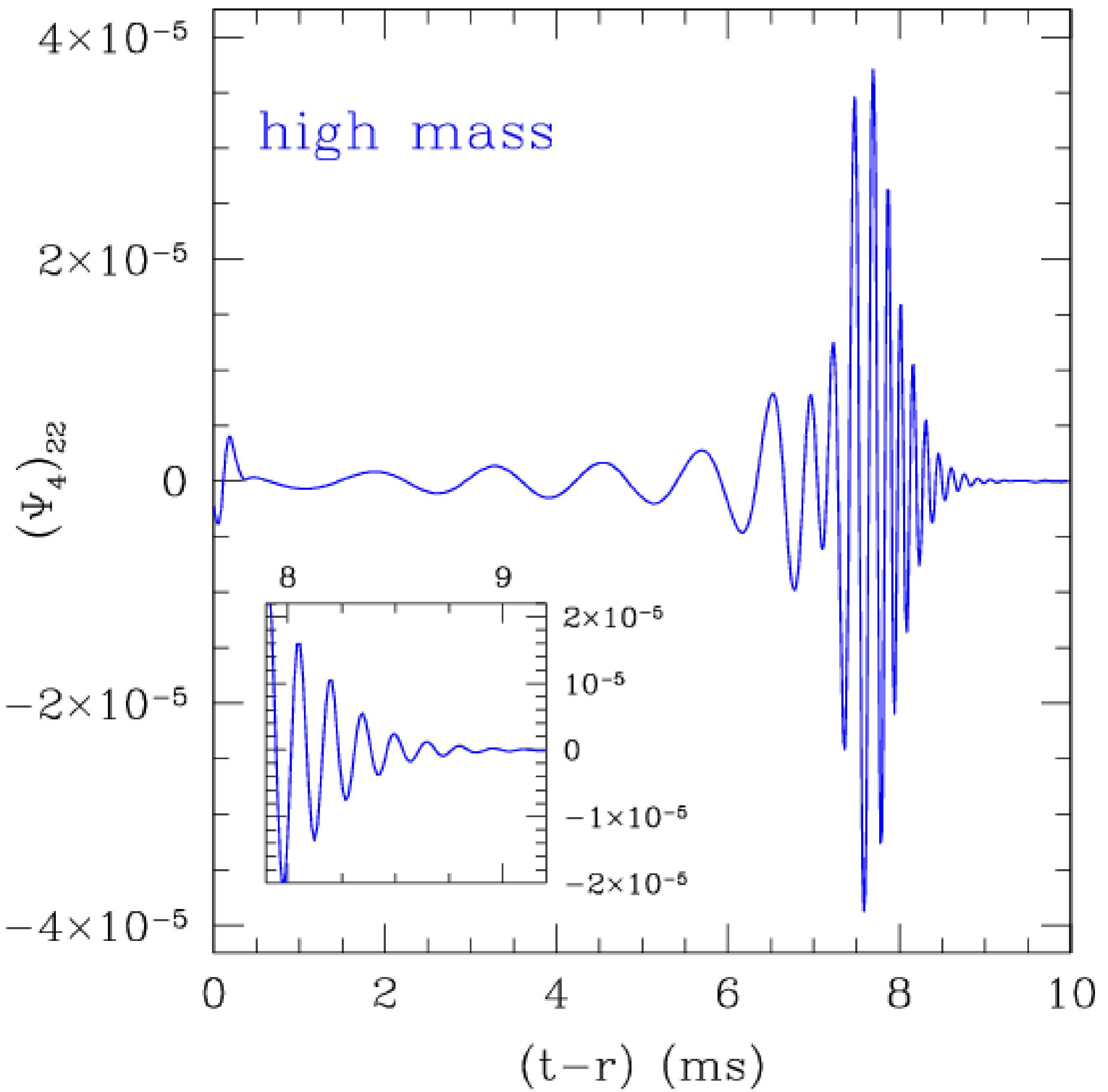


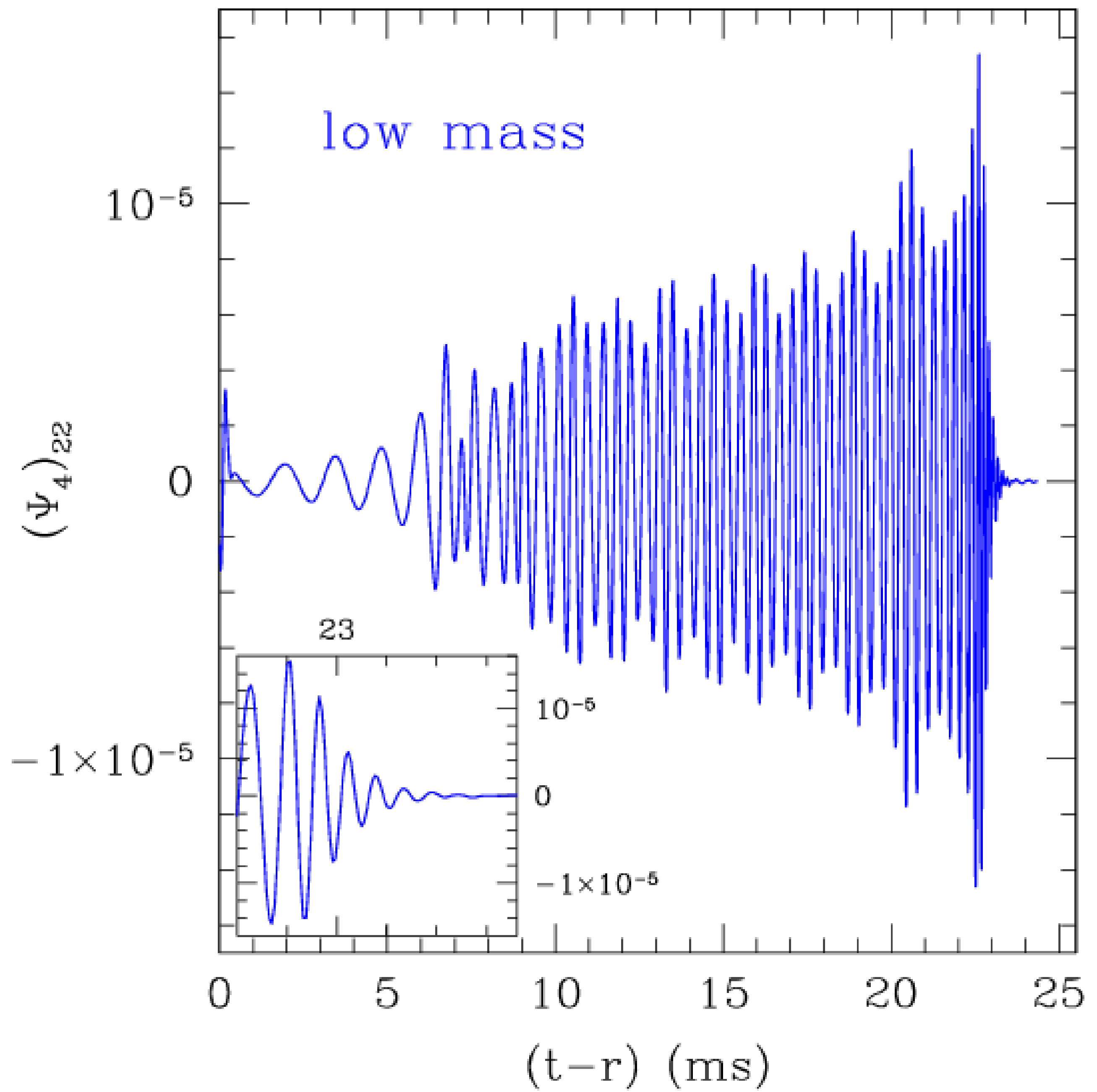
soon after the merge the torus is formed and undergoes oscillations

long after the merger a BH is formed surrounded by a torus

Waveforms: polytropic EOS

high-mass binary | low-mass binary

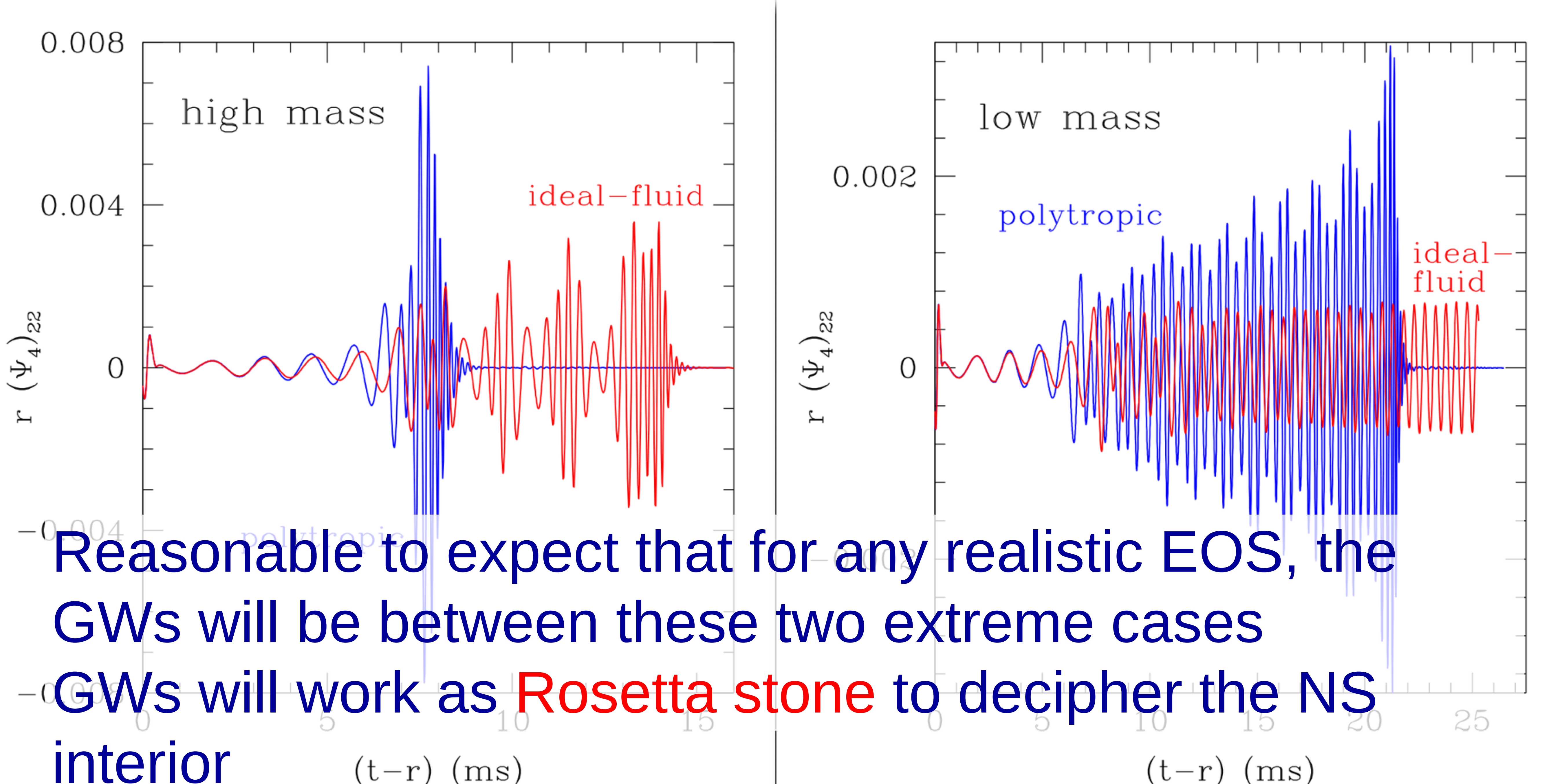




first time the full signal from the formation to a bh has been computed

development of a bar-deformed NS leads to a long gw signal

Imprint of the EOS: Ideal-fluid vs polytropic



Reasonable to expect that for any realistic EOS, the GWs will be between these two extreme cases

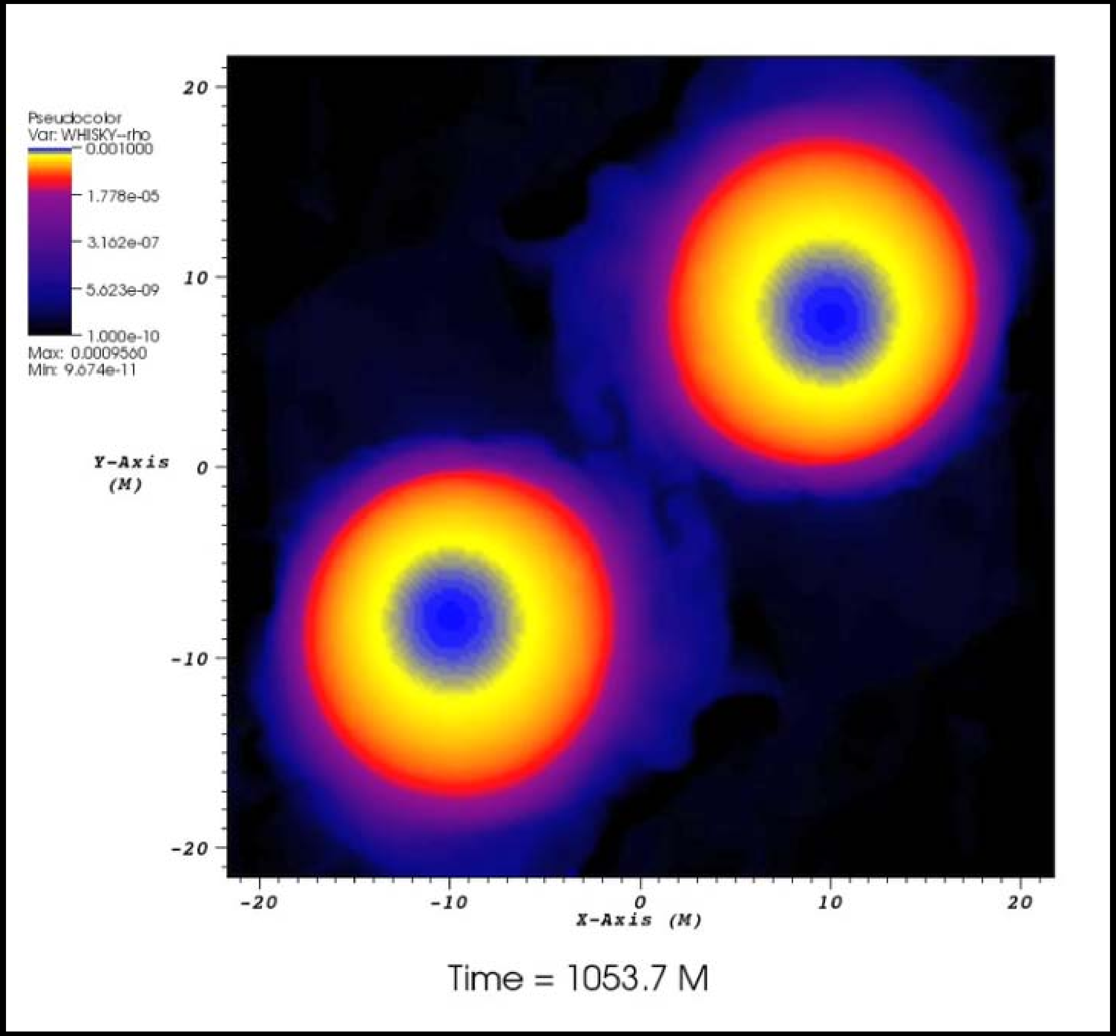
GWs will work as Rosetta stone to decipher the NS interior

After the merger a BH is produced over a timescale comparable with the dynamical one

After the merger a BH is produced over a timescale larger or much larger than the dynam one

Magnetized equal-mass binaries

Giacomazzo, Rezzolla, Baiotti, MNRAS Lett. 2009



Extending the work to MHD

We have considered the same models also when an initially poloidal magnetic field of ~10^{12} or ~10^{17} G is introduced

The magnetic field is added by hand using the vector potential:

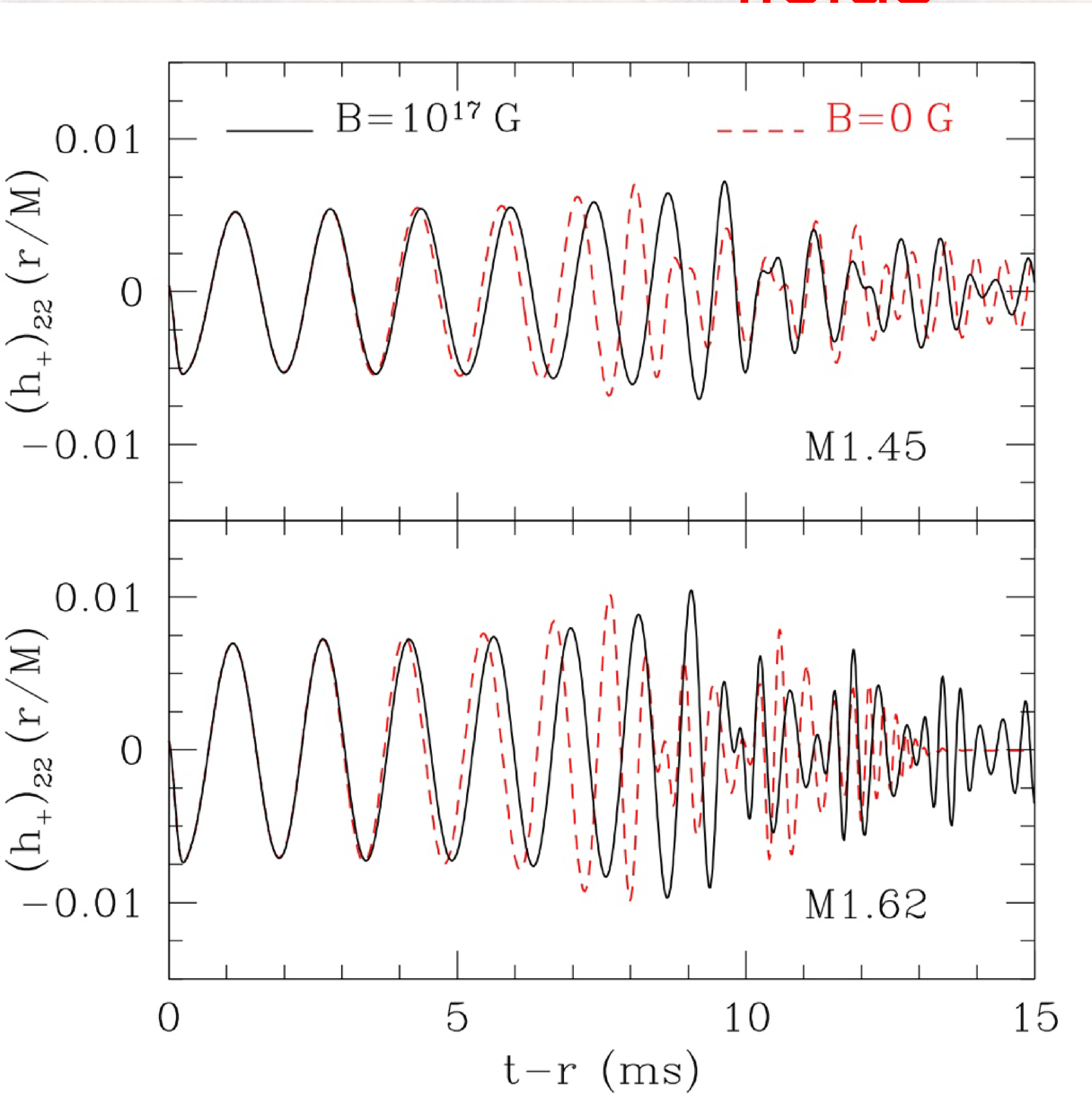
$$A_\phi = A_b r^2 [\max(P - P_{cut}, 0)]^n$$

where A_b and $P_{cut} = 0.04 \times \max(P)$ are two constants defining respectively the strength and the extension of the magnetic field inside the star. n=2 defines the profile of the initial magnetic field.

$$B = 0,\ 10^{12},\ 10^{14},\ 10^{17}\ \mathrm{G}$$

The initial magnetic fields are therefore fully contained inside the stars: ie no magnetospheric effects.

Waveforms: comparing against magnetic fields



Comparing against magnetic field strengths the differences are much more evident:

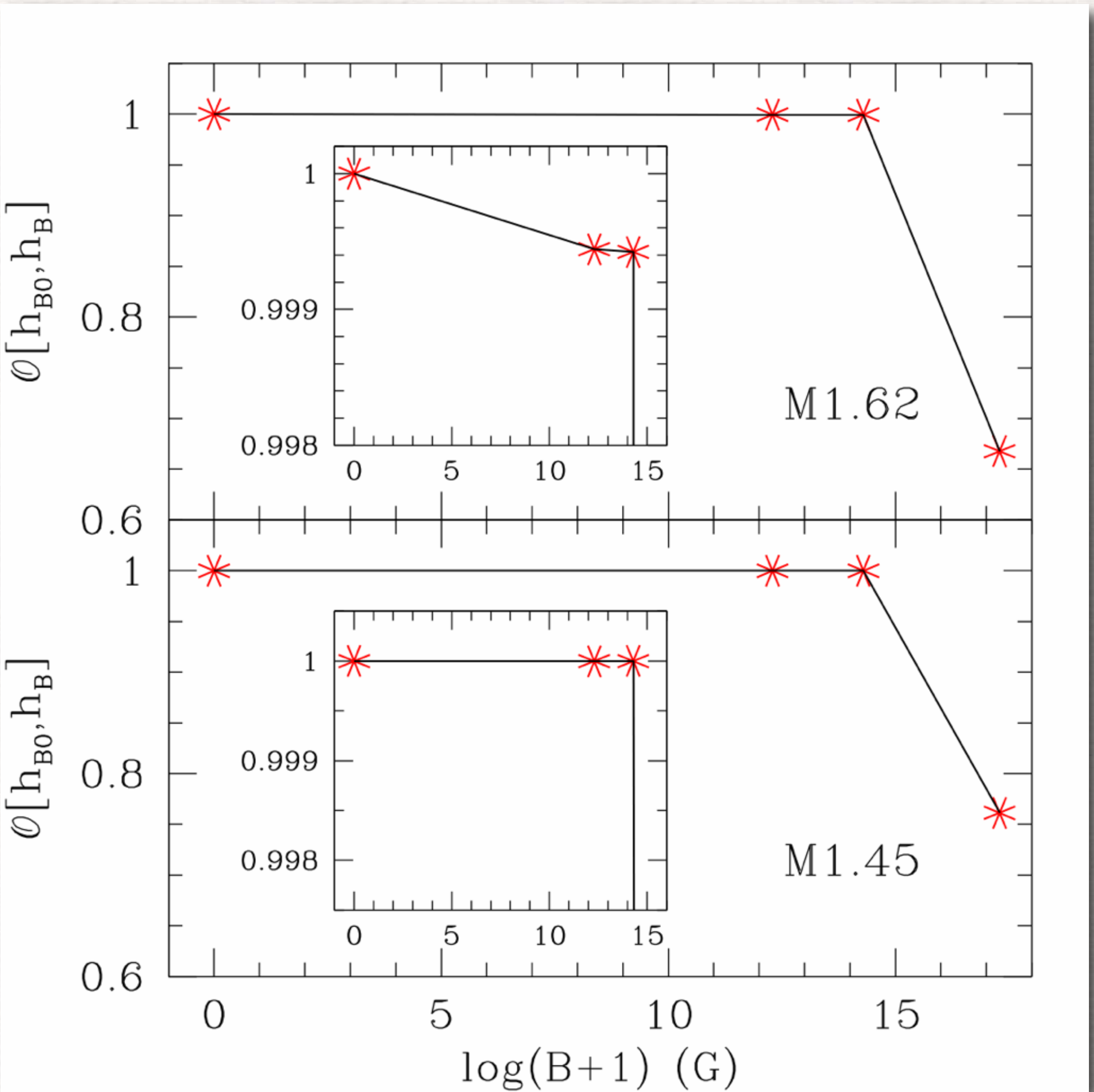
- the post-merger evolution is different for all masses (and essentially also for all MFs); strong MF delay the collapse to BH
- the evolution in the inspiral is also different for such large MFs

This confirms Anderson et al (2008). Is this true also for smaller MF

Understanding the dependence on MF

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the overlap

$$\mathcal{O}[h_{B1}, h_{B2}] \equiv \frac{\langle h_{B1} | h_{B2} \rangle}{\sqrt{\langle h_{B1} | h_{B1} \rangle \langle h_{B2} | h_{B2} \rangle}}$$



where the scalar product is

$$\langle h_{B1} | h_{B2} \rangle \equiv 4\Re \int_0^\infty df \frac{\tilde{h}_{B1}(f)\tilde{h}^*_{B2}(f)}{S_h(f)}$$

In essence, at these res:

$$\mathcal{O}[h_{B0}, h_B] \gtrsim 0.999$$

for $B \lesssim 10^{17}$ G

Because the match is even higher for lower masses, the influence of MFs on the inspiral is unlikely to be detected!

Animations: Koppitz, Giacomazzo, Rezzolla

Note that the torus is much less dense and a large plasma outflow is starting to be launched. The evolution has been stopped because of excessive div-B violations

Typical evolution for a magnetized binary

$$\mathrm{Ideal-fluid}, M = 1.65\, M_{\odot},\ B = 10^{12}\,\mathrm{G}$$

Conclusions

- Numerical relativity has made huge progress over the last few years; problems that were unsolved for decades are now well understood

- GWs from BNSs are much complex/richer than from BBHs: can be the Rosetta stone to decipher the NS interior

- The simulation of BBHs is well understood and most interesting physics is known; higher precision is important for current searches for gravitational waves.

- Much remains to be done to model realistically BNSs, both from a microphysical point of view (EOS, neutrino emission, etc) and a from a macrophysical one (instabilities, etc.)